[illegible]ntaxis [illegible]

[illegible] Or, A

CONSTRV[illegible]

Of the harder Pro[illegible] in

GEOMETRY:

With so much of the

CONICKS

as is therefore requisite,

And other more ordinary and usefull *PROPOSITIONS* intermixed:

And *TABLES* to several purposes.

The Contents follow in the ensuing leaf.

By *THO. GIBSON.*

Virgil.

Tempora dispensant usus & tempora cultus.

LONDON,

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and are to be sold at his Shop at the

Green Dragon in *St. Pauls*

Church-yard, 16[illegible].

CONTENTS.

CONTENTS.

TO

TO THE READER

ALthough no Book can be ſo copious, as wherein every Reader may be furniſhed with every thing which he looks for, yet there is ſeldome any thing like a Book which may not afford the Reader ſomething which he lookt not for. The hope that I may do ſo, may be taken for the reaſon why I writ *this*. But why I writ *thus*, that is, *Analytically*, *Valeſius*, *Lib. de Philoſoph. Sacr. pag*. 8. ſhall anſwer for me. *Si quis velit reſolutivum tenere ordinem, qui diſcentium naturæ magis ſe accommodas,*

modat, & petitionibus minus indiget, quia incipit à poſtremis de quibus primùm omnium continget dubitare. That is, If any one would hold the Reſolutive order, which more accommodates its ſelfe to the nature of Learners, and leſſe needs Petitions, becauſe it begins from the laſt things, of which principally men happen to doubt.

The method here uſed is the ſame as in Maſter *Harriot* in ſome places, that is, in ſuch Æquations as are propoſed in numbers. And as in *Des Cartes* in ſome other places, that is, in ſuch Æquations as are Solid, and not in numbers. Not that the Book is taken out of them, much leſſe that it proceeds continually with them, but disjunctly, as I thought fit to intermix them among other things which are not in them.

I ſhall uſe no arguments to commend the *Mathematiques*, or prefer them before the *Dogmatiques*, for this is but to write in praiſe of *Hercules*.

Yet this may be ſaid of them, that although

though ſome *bodily* exerciſes conduce more to *health*, and ſome *mentall labours* more to *wealth*, yet nothing affords the minde more *pleaſure*, or more *profit*, with leſſe repentance.

And therefore in a dull ſolitude, or vacancie of buſineſſe (both which may happen to Gentlemen) theſe are amiable company, which yield a delightfull and innocent expence of leaſure.

As for that Queſtion (which is frequent) *What profit is in theſe hard Studies?* it needs no anſwer, becauſe it imports the *ignorance* or *idleneſſe* of the Asker, or rather both.

For firſt (and which may be reprehenſive to many Writers, that muſt not be called Authors, which of late have brought up a faſhion to write in a *Quarying* way) he declares his *ignorance*, otherwiſe he needs not aske that which he knows.

Secondly, If ſome Meats were recommended to a man, with which he is yet unacquainted (otherwiſe they needed not recommendation) if he ſhould firſt aske

whether

whether they be good or not, that is, whether they would pleaſe his palat and ſtomach? the Queſtion is abſurd, for he cannot know that untill he have taſted and digeſted. So the idleneſſe of the asker may from hence be diſcovered.

Nor is there any *profit* to be gained by any *Science*, except the *Science* be firſt gained by *induſtry*.

Beſides, to think others, who being once entered herein, ſhould delight ſo much in them, as to make them a ſtudy all their life, if there were no profit in them; or if it were ſo, neverthefeſſe to recommend them to others, ſignifies another bad quality or two, which I forbear to name.

As for the difficulty of theſe Sciences, I muſt confeſſe that the firſt Aſpect of them may ſeeme uncouth and horrid (*Radices doctrinæ amaræ ſunt, fructus tamen dulces*) yet there is no reaſon why he ſhould be deterred hereby, and not rather animated with deſire to go as far as another, or elſe with ſhame to thinke there

there ſhould be ſo many Books in the world, eaſie to others and uſefull, but to him not underſtood, and therefore uſeleſſe.

In this following Treatiſe, my chiefe care hath been to render it all intelligible to every Reader, and I doubt not but it will prove ſo to every diligent one.

The Symbols and Characters herein uſed, are ſuch as have been long accepted in the world, without any innovation or fancie of my own, for although every Writer hath equall liberty herein, to adde or alter, as he ſees (or rather thinks) fit, yet in my opinion, we ought not to do this without conſiderable cauſe, or a kinde of convenience equipollent to neceſſity: for without doubt, he that increaſeth theſe, increaſeth his Readers burthen, eſpecially if ſuch increaſe be needleſſe.

I leave the reſt to the Reader to cenſure as he findes cauſe, and it is in vain to do otherwiſe, for in theſe demonſtrable things

things, the Readers *detection* of any error (of judgement) will be acceptable even to him that writ it, if he be civilly acquainted with it, but the said Readers *detraction* cannot here hurt any one but the Reader.

T.G.

ERRATA.

PAge 68. line 2. read $eee = ccc$. p. 70. l. 20. r. $cccc^3 = 729$. p. 76. l. 21. r. $3\surd 5 = \surd 45$. p. 96. l. 26. r. *Proposition*. p. 113 at Section 4. dele *Eucl.* 12. 7. and at Section 5. write *Eucl.* 12. 7. p. 124 & p. 129. in the Diagram the letters *x* and *y* are to be ſupplied upon the Diameter *a c*, ſee p. 128. l. 19. p. 134. l. 16. r. *Umbilicus*. p. 138. l. 8. r. *fk, g o, tl, &c.* p. 151. l. 1. r. $z = 132$. p. 178. l. 28. r. *for practiſe.* p. 183. l. 1. r. *right angles*. p. 185. the ſame diagram as in p. 183. ſhould be uſed. p. 187. l. 13. r. *Circle wno.* & l. 14. r. *n s*. p. 199. l. 7. dele *of*. p. 203. l. 1. dele *the*. p. 205. l. 17. r. *the arch f b.*

Preface.

ALthough in generall the *Mathematiques*, and especially the *Analitiques*, are easier in the beginning then proceeding (for the hardest is reserved for last) the *Principles*, *Petitions*, and *Definitions* also, seldome meeting any opposition, being (for the most) *first sight lessons* to all: yet I have thought fit, for some mens sake (who expect it in all Books) to premise some initiary things so easie, and so well known already, as must be received by every one. Neverthelesse, that it may not seeme trifling to the already knowing party, I will not be ample.

Common Sentences.

I. Greater.

1 The whole then the part.
2 Equall to that which is greater.
3 Greater then that which is greater.

 4 Neither

4 Neither lesse nor equall.

5 Double } Multiplex } of the whole.

6 Greater then that which is equall, *Euclid* 1.16.

7 Any thing commune or equall added to greater or to lesse, is greater then it. *Euclid* 4.17.

8 Where the parts are greater then the parts, the whole is greater then the whole.

9 Of two things, that which hath greater proportion to a third.

II. Lesse.

1 The part then the whole

2 Equall to the lesse.

3 Lesse then the lesse.

4 Neither greater nor equall.

And so by a way contrary to the former may be formed all that's *Lesse*.

III. Equall things.

1 That which is commune to two others, is equal to it selfe, as in *Euclid* 1. 5. an angle is commune, and in *Euclid* 1. 7. 8. 9. 10. 11. 12. a side is commune.

2 Those which are equall to the same thing.

3 Which are equall to equall things.

4 Which are equall to nothing.

5 The whole of equall things, added to equall. *Ax*.2.

6 The remain of equall things taken from equall things.

7 The whole of equall things added to a commune thing. Or *contra*. *Eucl*. 1.6.9.10.11.12.
8 The remain of equall things when a commune thing is deducted.
9 Verticall angles.
10 The Rectangles of the Means and Extreams.
11 Things which agree among themselves. *Ax*. 8. this last is proper to Geometry.
12 That which is not unequall, that is neither greater nor lesse: this is proper to homogeneals, for heterogeneals admit no comparison.
13 The whole to all the parts together.
14 The halves of the whole.
15 Whose halves are equull.
16 Whose parts are equall in Number and Magnitude.
17 Whose Doubles } are equall.
Whose Equimultiplices } are equall.
18 If the parts be equall to the parts, the whole is equall to the whole.
19 If nothing else be equall besides the thing supposed, that thing is equall.
20 Which have the same proportion to the same thing.
21 Those to which the same thing hath the same proportion.
22 Of four proportionals, if the first be equall to the third, the second is equal to the fourth.
23 If there be twice three Magnitudes, which taken by two and two are in the same proportion,

if (of æquality) the first be equall to the third, the fourth is equall to the sixt. *Euclid* 5. 20.

24 If there be twice three Magnitudes, which taken by two and two, are in the same proportion, and the proportion be perturbate, if the first be equall to the third, the fourth is equall to the sixt. *Euclid* 5. 21.

IV. Agreeing things.

1 Are such as are equall, and of the same kind.

V, Unequall things.

1 Greater or lesse,
2 The whole and the part.
3 The whole, when a commune thing is added to unequall things.
4 The whole, when an equall thing is added to unequals. *Euclid* 1. *Ax.* 4.
5 The Remain, when a commune or equall thing is taken from unequall things.

VI. Double.

1 The double of the halfe.
2 Two equall things taken together are double to one of them.
3 The double of that which is equall.
4 That which is equall to the double,
5 If the parts be double to the parts, the whole is double to the whole.

6 The

The 6. proportion of like figures to their ſides of like proportion, *Euclid* 6. 19.

VII. Halfe.

1 Is the halfe of that which is double.

2 That which is equall to one of two equals, is the halfe of them together.

3 The halfe of an equall thing.

4 That which is equall to the halfe.

5 The proportion of like ſides to the proportion of like figures.

VIII. A thing is.

1 If nothing elſe which can be propoſed is the thing, then this which was propoſed *is*. *Or*,

2 If any thing elſe beſides that ſuppoſed be put, and an impoſſibility follows, then this which was ſuppoſed *is* that which was ſought.

3 If that which is ſuppoſed be nothing elſe, then it *is* what was required. *Or*,

4 If this which is ſuppoſed being put for any thing elſe, an impoſſibility follow, then it *is* what was required.

5 That which neceſſarily follows from that which *is*.

6 Which put for not in being, there follows an impoſſibility.

IX. Something.

1 Is that which if any thing be added to it, it is

more, or if any thing be taken from it, it is less, or to which if nothing be added, it is the same.

2 To which if lesse then nothing be added it is lesse.

3 If lesse then nothing be substracted, it is more.

4 Which multiplyed by something is more.

5 Which multiplyed by nothing is nothing.

6 Which multiplyed by lesse then nothing, is lesse then nothing.

6 Or that which divided by
Something, is lesse.
Nothing, is nothing.
Lesse then nothing, is lesse then nothing.

X. Nothing.

1 Is that which added to, or taken from something, or lesse then nothing, leaves it the same it was: and multiplying or dividing something produceth nothing, but takes the thing quite away.

XI. Lesse then nothing.

1 Is that which added to something makes it less.

2 Which substracted from something makes it more.

3 Which added to lesse then nothing makes it still lesse.

4 Which taken from lesse then nothing makes it more.

5 Which multiplyed by something gives lesse then nothing.
6 Which multiplyed by lesse then nothing, produceth something.
7 Which divided by something, makes lesse then nothing.
8 Which divided by lesse then nothing, makes something.

XII. Unity.

1 Is that to which if unity be added it is doubled.
2 From which if unity be taken, it is nothing.
3 If more then unity be taken, it is lesse then nothing.
4 If lesse then unity be taken, it is lesse then unity.
5 If lesse then nothing be taken, it is more then unity.
6 Is that which cannot be multiplyed or divided by unity without remaining the same.
7 Is the difference of the two greater sides of a rectiline rectangle triangle: or may be so by reduction of the sides to lesser numbers. *vide Corol: ad Cap. 7 Prob. 3.*

Propo-

Propoſitions of EUCLIDE, fit to be known to the ANALIST.

In the firſt Book *Prop.* 6, 13, 14, 15, 18, 19, 28, 32, 43, 47, 48. In all 11.

In the ſecond book all but the eleventh, and laſt, in all 12.

In the third, *Prop.* 14, 20, 22, 31, 32, 35, 36, in all 7.

In the fift Book, *Prop.* 15, 16, 17, 19, 24, 25, in all 6.

In the ſixt Book, *Prop.* 2, 3, 4, 6, 7, 8, 13, 14, 16, 19, 24, 31, in all 12.

Many more propoſitions out of theſe and the remaining Books might be uſefull: But theſe 48 laſt reckoned are ſuch as (in my judgement) ought chiefly to be read, and remembred, for aſſiſting to attain and reſolve Æquations.

Now whereas it is ſaid in the enſuing Chapter, that vowels are put for things unknown, or ſought: and conſonants ever for known things, it is to be noted that in a Scheme which imployeth almoſt all the Alphabet theſe are promiſcuous.

But in abbreviation or demonſtration, whereſoever one ſingle letter is put (or ſuppoſed to be) equal to any line or number, although the ſame letters which before deſigned the Diagram

be,

againe uſed herein, yet in a different acception; For whereas in the Diagram they ſignified points. now they ſtand for lines or things; And evermore the conſonants ſignifie things given or known before; and the vowels (although all preſent) are ſuppoſed equal to things which are not yet known, but about to be found.

Onely the vowel *o* is ſeldome uſed in this ſenſe, becauſe it is uſurped in another, that is to ſignifie nothing. As $a - b = o$. ſignifies that *a* want *b* is equal to nothing: or that *a* is equal to *b* where the vowel *o* ſtands for a cipher, that is nothing. On the other ſide the Greek vowal *y* is uſually put for any unknown quantity.

Definitions.

Definition I.

The unknown Quantity of any æquation is called generally *Poteſtas*; or a *Power*, *Quantity*, or *Terme*.

Definition II.

A *Rectangle* is in numbers the *Product* of two numbers multiplying one another.

In Geometry it is the *Area*, ſpace, or content of a right angled quadrangular figure, made alſo by multiplication of two lines, which are called the ſides; of which one is the meaſure of the breadth the other of the length.

Definition

Definition III.

A *Rectangled Parallelepipedon* is the product of a Rectangle multiplied by a right line or number: And if that line or number and the length and breadth of the Rectangle be severally equal it is a Cube, or Die.

Definition IV.

A *Prisme* is a Solid contained within five superficies of which three are Quadrangular, and the other two being opposite, are triangular: Or it is like the top of an ordinary English house cut off by a Plane passing through or parallel to the Eaves.

The rest of this kinde I shall not define here but referre the Reader to *Euclid*.

The names of the Potestates or Powers.

1 The first *Power* is called a Side, or Root: The later word *Root* is most used here; and it is signified thus, *a*.

2 The second *Power* is called a *Square*, and is thus written, *a a*.

3 The third is called a Cube, and is thus written, *a a a*. or sometimes for brevity, a^3

4 The fourth, a *Biquadrat* or squared square, anciently a *Zenzi zenzick*, figured thus ʒʒ now thus, *a a a a*, or for brevity, a^4.

5 The

5 The fift Power is called a *Surſolid*, and is written thus, *a a a a a*; or briefly thus a^5.

6 The ſixt, a ſquared Cube, or *zenzicube*, written thus, *a a a a a a* or a^6.

7 The ſeventh, a ſecond *Surſolid*, and is written *a a a a a a a*, or more ſhort a^7.

8 The eight ls called a ſquared ſquare ſquared, or *zenzi zenzi zenzi zenzick*, and is written *a a a a a a a a*; or thus a^8, &c.

Conſectary I.

Hence it is manifeſt that theſe powers uninterrupted, are in continuall proportion, the proportion of them being as *a*, to unity: or the converſe.

Conſectary II.

It is alſo here plaine, that every *Power* hath ſo many dimenſions, as the letters, with which it is written. For a^4 being written with foure letters, if one letter ſtand for one dimenſion, that is length or breadth, the other three ariſe by three ſeverall Multiplications, and every Multiplication addes a dimenſion, in this ſenſe.

A Table of the Powers of the Digits.

1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

In the former Table, the Digits at the top 2, 3, 4, &c. Shew the Columnes of the ſecond, third, fourth, &c. *Powers.*

The Digits, at the left ſide, ſhew the ſeverall Roots or firſt Powers, and their proportion to Unity. All the reſt is evident.

And now becauſe towards the end of this litle Treatiſe, I ſhall happen to ſpeak once ortwice of Arithmetical Calculation;The Reader may hereby underſtand, that ſuch Calculations are uſually (and moſt eaſily) performed by numbers aſſumed in Arithmetical proportion, called Logarithmes; of which I intend to ſay nothing, ſuppoſing any Reader converſant about ſuch things wherein I uſe them, cannot be ignorant of them and their uſe.

Such as be, ſhould read that firſt, of which they need no better (nor other) inſtructions then ſuch as they may have in Mr. *Norwoods* doctrine of Triangles; which is a Book not very deare.

But to ſuch as have not that, theſe following directions may be of ſome uſe.

1 In every Spherical triangle which hath one right angle, or one ſide a quadrant, all the other five parts(for every triangle hath ſix in all, that is three ſides, and three angles) are called *Circular Parts.*

2 Of theſe Circular parts, if any two be given, the reſt, that is, any one of the reſt,may be found ta one operation.

3 For thoſe two are either adjacent, or remote, or oppoſite: if adjacent,& the part required be alſo adjacent (or touching) to one or either of them, then that one ſo touched on the one ſide by a part given, and on the other ſide by a part required, may fitly be called

The Middle Part

And it is a demonſtrated truth, that,

As the tangent of the known part adjacent,
is to the right ſine of the middle part;
So is the Radius or Semidiameter,
to the tangent of the unknown, or required part; being alſo adjacent to the middle part, as before.

And therefore. if inſtead of the naturall ſines and tangents, the Logarithmes be uſed,they being in Arithmeticall proportion, the ſumme of the two middle termes is equal to the ſumme of the two extreames; And ſo here, the ſine of the middle part *plus* Radius is equal to the tangent of the of the adjacent part known *plus* the tangent of the part required.

I hope the word *plus* needs no interpretation.

Note 1. It is notwithſtanding to be ever remembred that every of the five circular parts muſt be conſidered two wayes; that is whether it be contiguous to the right angle, or quadrant; if ſo then all before is right and unalterable.

But if not ſo,that is,if ſome other part lie betwixt them, then all that hath been ſaid of their ſines and tangents,

tangents, which were then supposed contiguous, must be performed by the sines complements and tangents of the complements of such of the parts respectively as are remote from the right angle or quadrant.

4 If now the two parts given be remote, and the part required lie betwixt them, then make the part required the middle part, and it may be found as easily as in the former case.

5 If the two known parts be contiguous, and the part required adjacent to neither of them, but opposite to one of them, then (working still by Logarithmes) make the part required the middle part, and then the sine of the middle part *plus* Radius, is equal to the sines complements of the opposite parts given, if therefore from those two sines complements added, be taken Radius, the rest is the sine of the thing required.

Note 2. It is further to be noted that the sines complements of those parts which by the former note are accompted complements, are the sines of the things themselves.

Example.

In the triangle *z p s* let *p* be the Pole, *s* the Sun and *z* the Zenith *z p s*, the hour from noon in Winter, or the hour from Midnight in Summer; *p z s*, the Azimuth from the North; and *z s p*, the angle of position: and *s z* be 90 Deg. as at Sun-rise. Then

Then firſt, ſine *z plus* Radius, is equal to tangent complement *z p plus* tang. *s*.

Secondly, ſine compl. *p plus* Radins is equal to tang. compl. *p z plus* tang. compl. *p s. per Not.* 1.

Laſtly, (by the 5 direction) ſine compl. *s p plus* Radius is equal to the ſine of *p z plus* the ſine complement of *z*; becauſe *p z* is accompted a compl. and *z* not ſo. *Note* 1.

And if inſtead of a quadrant (as *z s*) there were a right angle, at *p*, or at *z*, or at *s*, the aforegoing directions ſerve.

If there be neither right angle nor quadrant, there muſt be two operations to do this, having firſt ſuppoſed a circle to paſſe from one of the angles to cut the oppoſite ſide (produced when need is) at right angles.

This perpendicular circle ſhall fall ſometimes within the triangle, ſometimes without.

Within, when the other angles are both obtuſe: Or both acute, as at the hours between ſix and noon are the angles at *p* and *s*.

In

In letting fall the perpendicular, Mr. *Norwoods* advice is to do it.

1 *From the end of a side given, being adjacent to an angle given, let it fall opposite to that angle.*

2 *And touching in some part the side required.*

3 *And opposite (if it may be) to the angle required.*

One of the most difficult cases in oblique angled sphericall triangles is this.

In the triangle $p\ z\ s$. Let there be given,

comp. Elevation $p\ z = 38.28'$
comp. Declinati. $p\ s = 66.29'$
comp. Azimuth $p\ z\ e = 70.\ 0'$

To find the complement of the Suns Altitude $z\ s = 75.45'$ having produced $s\ z$, and from p, let fall $p\ e$, perpendicular to it: then,

First, making $p\,z\,e$ (the middle part) s.c. $p\,z\,e$. *plus* Radius	195340517
From which taking the tangent of the Elevation, t.c. $z\,p$.	100999135
Remains tang. of $z\,e = 15.12'$ t. $z\,e$	94343382
Secondly, to sine compl. $z\,e$	99845347
Adde sine of Declination, s.c. $p\,s$	96009901
The summe of them is	1958 55248
From which taking sine Eleva. s.c. $p\,z$	98937412
Remains (the sine of 29.27′) s.c. 60.33′	96917696

To whose comple. 60.33′ adding $z\,e = 15.12'$, the sum is 75.45′ the thing required.

Whereas, if the Azimuth it selfe $p\,z\,s$ were 70d, then $z\,e$ being taken from 60.33′, the rest 45.21′ is the Suns Altitude.

CHAP. I.

An explanation of the Characters and Symbols used in this Work.

FIrst, One single letter of the Alphabet is usually put for any quantity whatsoever, as well Line as Number, whether known or unknown.

But for the most part, where any quantity is sought

ſought, there *a* or ſome other *Vowel* is put for it; and the other Quantities known, are ſignified by *Conſonants.*

Theſe letters are multiplyed one into another by joyning them together without any pricke or comma between, nor doth it import at all which is firſt or laſt written: for $b\,c\,d$, $b\,d\,c$, and $c\,b\,d$; are all one.

So *a* multiplyed by *a* produceth $a\,a$.
And *a* multiplyed by *b*, produceth $a\,b$.
And $a\,b$ multiplyed by *c*, produceth $a\,b\,c$.

The like of all others whatſoever, except Fractionall quantities; as, $\frac{ab+fg}{d}$ and $\frac{bc-fb+rc}{b+c}$

If the firſt of theſe were to be multiplyed by *d*, it is done by taking away the *d* under the line, and the product is $a\,b+f\,g$.

If the ſecond were to be multiplyed by $b+c$, it is done by taking away the Denominator $b+c$, and the product will be $b\,c-f\,b+r\,c$.

For all Fractions aſwell in Plaine as in Figurate Arithmetick, are nothing elſe but Quotients of one number divided by another; and are multiplied again by taking away their Diviſor, and line of Separation.

Diviſion is done in Figurative Arithmetique, moſt commonly by applying ſome line of ſeparation between the Dividend and the Diviſor.

So $\frac{a}{b}$ is a divided by b, And $\frac{a\ b\ c}{f}$ signifies that $a\ b\ c$ is divided by f.

But yet if the letter f had been found in the Dividend, the application of this line had not been necessary, for it might have been better done by taking away that letter out of the Dividend.

So $a\ f\ c$ divided by f quotient is $a\ c$
and $f\ f\ c\ c$ divided by $f\ c$ quotient is $f\ c$
by $f\ f$ quotient is $c\ c$
by $c\ c$ quotient is $f\ f$
by $f\ f\ c$ quotient is c
by $f\ c\ c$ quotient is f
by f quotient is $f\ c\ c$
by c quotient is $f\ f\ c$

And the like may easily be understood of all the rest.

Symbols of	Majority	$>$
	Minority	$<$
	Æquality	$=$
	Addition	$+$
	Substraction	$-$
	Root of a quantity	$\surd$

Proportionality continued ′ ″ ‴ ⁗
Proportionality disjunct ′ ″ ′ ″

So

So $b > c$ ſignifies b greater then c
$b < c$ .... b leſſe then c
$b = c$ .... b equall to c
$b + c$ .... c added to b
$b - c$ .... c taken from b
$\sqrt{72}$ ſignifies the ſquare root of 72, &c.

And b' c'' d''' f'''' ſignifieth that as b is to c, ſo is c to d, and ſo d to f.

Likewiſe b' c'' f' g'' ſignifies that as b is to c, ſo is f to g.

Theſe things before expreſſed are almoſt generally received: and uſed not only for brevity in writing, but perſpicuity in proving, as will be ſeen hereafter.

Note that whereſoever $-$ is not expreſſed, there $+$ is underſtood, though it be not expreſſed.

Alſo in Trigonometrie. I uſe, s. pzs, for the ſine of an angle pzs, and s.c. zp for the ſine of the complement of a ſide pz to 90. Alſo, t. zp and t.c. spz for tangent of zp and tangent of the complement of spz, &c. Alſo for Radius I uſe r.

If the ſigne of Addition, namely $+$ ſtand before any quantity, it ſhewes that quantity, to be more than nothing; that is ſomething.

But if the ſigne of Subſtraction, to wit $-$ ſtand before any quantity; it ſhews that quantity to be leſſe than nothing: or a want of the ſaid quantity.

Soe $+4$, ſignifies four of any thing: but -4, ſignifies a want of four, or four leſſe than nothing.

In ADDITION.

The addition of a want of any thing, is all one with the ſubtraction of the ſame thing.

So if to + 12 you adde — 5 it makes + 7
And if to + 12 you adde — 16 it makes — 4
But if to + 12 you adddde + 16 it makes + 28

In SUBTRACTION.

The ſubtraction of — is all one with adding +

So if from + 12 you ſubtract — 5 remain. is + 17.

And if from + 12 you ſubtract — 16 remain. is + 28.

Addition of + to + and Subtraction of — from — is all one with Common Addition and Subtraction. And generally for both.

In *Addition*, add the quantities together with the ſame ſigne.

In *Subtraction*, adde them alſo, but all the ſignes of that which is to be ſubtracted from the other, muſt be changed.

Example.

If to + 6 — 2 + 3, be added + 5 + 1 — 3 the ſum is + 6 — 2 + 3 + 5 + 1 — 3 = 10.

But if from + 6 — 2 + 3, be ſubtracted + 5 + 1 — 3, the remain is + 6 — 2 + 3 — 5 — 1 + 3 = 4. This Rule is generall, and generally known.

In

In MULTIPLICATION.

+ multiplied by + ever produceth +
+ multiplied by — ever produceth —
— multiplied by — ever produceth +

More Varieties there are not.

The quantities that are accompanied with these signes of + & — (in both Multiplyers being placed one under another, as in cōmon multiplication) must be multiplyed every one below into very one above, and then this work is done.

So if, $+bb+b-c$, be multiplied by $+f-g$ place them thus.

$$\begin{array}{l} +bb+b-c \\ +f-g \\ \hline \end{array}$$

Saying, $+f$ multiplyed into $+bb$ gives	$+bbf$
And $+f$ into $+b$ gives	$+fb$
And $+f$ into $-c$ gives	$-fc$
And $-g$ into $+bb$ gives	$-bbg$
And $-g$ into $+b$ gives	$-bg$
Lastly, $-g$ into $-c$ gives	$+cg$

Which added together is, $\left.\right\}$ $bbf-bbg+fb-fc-bg+cg$

Which is the true product.

In DIVISION.

If the line of separation doe not serve the turne that is, if any desire, (and it may be done) other-

 wise,

wise, it must then be by seeking what quantity may be multiplied by the Divisor to produce the Dividend.

So if $bb+bc-bf-bg-cg+fg$, were to be divided by, $b+c-f$, triall must be made what mixt quantity multiplying $b+c-f$ will produce $bb+bc-bf-bg-cg+fg$,

In which there is this of Compendium, that seeing the Dividend consists of six members, and the Divisor of three, the quotient must be of two; that is a *Binomial* only.

And because the quantity g is found in the Dividend, and not in the Divisor, it must be in the quotient.

The said quotient therefore must be one of these, $b+g$, $b-g$, or $g-b$.

It cannot be the first, for $+g$, into $-f$ would have produced $-fg$: but in the Dividend it is $+fg$, therefore it must be $-g$.

By the same reason it cannot be the last, as also because $-b$, into $+b$ produceth $-bb$, but it is $+bb$, in the Dividend.

The quotient sought, must therefore be $b-g$.

Some further Rule for saving labour herein might be given: but every one likes that best which he finds out himself. Nor is it my purpose to write a *Booke of Algebra*; but to premise so much of the Rudiments thereof, as the Reader may stand in need of in perusing the following Treatise.

Wherein

Wherein because Division is seldome needed; If I have a litle exceeded already, and shall a litle more in treating (but very briefly) of resolving some few Rooted Æquations, I shall aske the Readers pardon for both together.

CHAP. II.

Of Æquations.

AN Æquation is when one or more speciall quantities, are equal to one or more other speciall quantities, and written with the signe of equality betwixt them; As $aa = bb$.

This is called a simple Square Æquation. And bb, being a known square, the square root thereof being extracted, is equal to a. And that is the thing required.

But, $aa + ba = cc$, and $aa - ba = dd$, and lastly $-aa + ba = ff$; are all of them of that kind, which are called mixed æquations, because a (the thing required) is multiplied not only into it self, but into another known quantity, namely into b.

And note that this known quantity in all mixed æquations is called the *Coefficient*.

Note also that the three sorts of mixed æquations above expressed are all that can happen in quadratiques: And by some one of these, all Pro-

blems

blemes whatsoever transcending plaine Division, and not reaching Solids, are to be resolved by finding the Root *a*, according to these *Old Rules*.

In the First, $aa + ba = cc$.

Unto the quantity given namely *cc*, *adde the Square of half the Coefficient, it makes* $+cc+\frac{bb}{4}$ Which if it be in lines, may be reduced into one Square, *and from the side of that Square, take half the Coefficient, and the remainer shall be* *a*. Which was the thing desired.

In the second, $aa - ba = dd$.

Unto *dd* adde $\frac{bb}{4}$ as in the former, and the sum thereof being always in numbers a Square, or in lines to be reduced to a Square as aforesaid; *Unto the Root or side of that Square, adde halfe the Coefficient*, the Summe thereof shall be *a*, or the Root of the Æquation sought for.

In the last, $-aa + ba = ff$.

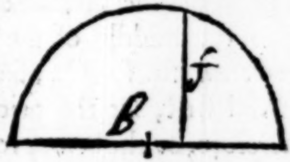
From the square of half the Coefficient, which is $\frac{bb}{4}$ *take the quantity given*, that is *ff* there will remain $\frac{bb}{4} - ff$, which being put into one Square, and the side thereof known: *If that*

side

side be either added to halfe the Coefficient, or subtracted therefrom, either the summe of that addition, or the remain of the subtraction, is equall to a.

For all Quadratique Æquations of this kinde (where aa the greatest unknown power is wanting) have two Roots, which being both together ever equall to the Coefficient, if upon the Coefficient, as a Diameter, a Semicircle be described, and the side of ff (the quantity given) be applied therein, perpendicular to the Diameter b, the two segments of b are the two Roots sought.

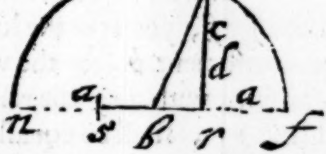
For in the Æquation $-aa+ba=ff$, it is by the 14th. of the 6th. of *Euclid*, as followeth.



$b-a' \quad f'' \quad f' \quad a''.$

Wherefore either Segment may be a, and the other will be $b-a$, and f a mean betwixt them.

Likewise in the two former Æquations, the worke may be effected Geometrically, & proved also by this present Scheme.



In which, as the figure intimates, the Perpendicular

dicular represents the side of $c\,c$ in the first Æquation, and the side of $d\,d$ in the second.

Draw a line $b\,g$ from the center b to the top of the perpendicular, the center b being first taken in the middle of the line b, to wit, of the *Coefficient*, for so it is usually called.

And first, let the pricked line be put for a, Therefore, by the before recited Proposition,

It is, $a + b\,'\;\;c\,''\;\;c\,'\;\;a\,''$. *Eucl.* 6.14.

And if (as the Rule prescribeth) to the square of halfe b you adde the square of c, the totall shall be the square of the line $b\,g$: By the 47th. of the first of *Euclide*,

If therefore from the line $b\,g$, or (which is all one) $b\,f$, you take the line $b\,r$, which is half the Coefficient b (for the whole Coefficient b, is the same with $s\,r$) the rest, namely the pricked line $r\,f$, shall be equall to a. For,

$$r\,f = n\,s = a.$$

In like sort concerning the second Æquation,

$a\,a$

$aa - ba = dd$: If according to the Rule, you adde the ſquares dd, and $\frac{bb}{4}$ together, it gives the ſquare of the line bg, to the Root of which, to wit, bg, if you adde halfe the Coefficient, to wit, br, or bs, the ſum ſhall be fs or nr, either equall to a. And then, as nr, that is a, is to d, ſo is d to rf, or $a - b$, as it ought to be.

I intend anon to write ſomething of *Extraction of Roots*, according to the generall Method of reſolving all manner of Æquations of *Powers*, how high or compoſed ſoever. I do not mean to exemplifie them any further then the *Cubique* order. There are Authors enough, whom they that deſire the full of that Artifice, may at their own leaſure in Books conſult.

And now becauſe I ſhall herein make ſome uſe of Æquations, though not higher then Cubiques, or at the moſt the Biquadratique order: I think fit to admoniſh the Reader, that in putting a alwayes for the thing ſought, and working therewith, as if it were known, quite through as the queſtion requires, he ſhall at laſt come to an Æquation, but it may be ſuch a one as wants reducing: of which a little.

REDUCTION

Of Æquations is done by adding all that's neceſſary, or ſubtracting all that's not neceſſary on

both

both sides the signe of æquality: Or by subtracting contradictories if they happen on one and the same side, untill the Æquation, purged of all unnecessary members, remain with all that's absolutely known on one side, equall to (as little as may be) all that's unknown on the other side.

One example of this shall serve as followeth:

In the Æquation

$$aa - ba + dc + ba = gg + ba - dc.$$

To reduce this, you must remember what hath been said before; that the taking away a *Want* of any thing, is all one with the addition of that thing.

Therefore seeing there is on the first side a *Want* of ba expressed by $-ba$, if you take away that $-ba$, you thereby adde ba on that side.

Wherefore that it may still be an Æquation, you must adde ba on the other side also.

Then it will be,

$$aa + dc + ba = gg + 2ba - dc$$

Again, subtract ba on each side, then it is,

$$aa + dc = gg + ba - dc$$

Once more, subtract ba on each side, that you may bring it to that side where aa stands.

Then it is,

$$aa - ba + dc = gg - dc.$$

Lastly

Lastly (that the Consonants, or known things may come all on one side) subtract $d\,c$ on each.

Then it will be,

$$aa - ba = gg - 2\,d\,c.$$

Take the Rectangle $2\,d\,c$ out of the Square $g\,g$, and let the rest be a Square, namely $f\,f$.

Then it is reduced,

$$a\,a - b\,a = f\,f.$$

Having gone a little about, only for exercise of them that are quite unskilfull herein, now they shall see this Reduction might have been quickly done another way, that is, seeing in the Æquation

$$aa - b\,a + dc + b\,a = gg + b\,a - d\,c$$

There are in the first part Contradictories, to wit, $-\,b\,a$ and $+\,b\,a$, they (destroying one another) might be taken away both at once,

So it will be,

$$aa + dc = gg + b\,a - d\,c.$$

Then if you subtract $d\,c$ and $b\,a$ on both sides, it will be reduced to

$$aa - b\,a = gg - 2\,dc,$$

as it was before. And $g\,g - 2\,d\,c$ being put into one Square $f\,f$, the Æquation

$$aa - ba = f\,f,$$

may be resolved as the æquation $aa - b\,a = d\,d$

was

was, by the second Rule for plain Æquations, a little before expressed.

And as here the Reduction was made by Addition and Subtraction only, so sometimes it is made by Multiplication, sometimes by Division; in both or either of which, this is generall: that *Whatsoever is done to any one Member, must be done to every Member quite through the Æquation.*

CHAP. III.

Of the resolution of Æquations, according to the generall Method composed by Mr. Tho.Harriot.

ALthough (having before shewed Rules for all sorts of mixed squares) it may seeme preposterously done hereafter to speake of Simple Squares; yet forasmuch as I pretend not much to Method or Order, and because the generall Method of Mr. *Harriot* begins with Squares, I will do so, but only with one Example. That is; Let there be an Æquation of $aa = ff$.

Or let it be exhibited in numbers, $aa = 69169$

First, take notice that all Squares whether simple or mixed in Numbers, are to be marked with points, the first alwayes over the place of Unity or unities, and so successively every Binari or second figure.

Cubes

Cubes with every ternary figure.
Biquadratiques with every quaternary.
Surſolids, every quinqnenary, and ſo forwards.

This ſquare number ſo pointed is 69169

In which becauſe there are three points, there are three figures in the Root.

So that *a* being a ſingle letter cannot fitly repreſent that Root, but ſome trinomiall, as is $b+c+d$ ſhould be put equall to *a*, and the ſquare thereof ſhould be equall to aa, or 69169.

But becauſe it may be done aſwell by adding the Gnomons, that is repetition of the ſecond working, (as they are commonly called) ſo often as the points are more then two; a *binomiall* will ſerve (with leſſe trouble) to do the ſame.

Let that *Binomiall* be $b+c$.
And put $b+c=a$.
Their Squares ſhall be therefore equall.
That is, $bb+2bc+cc=aa$.
That is, $bb+2bc+cc=69169$.

The Reſolution.

The homogeneall number given 69169
Firſt ſingle Root $b=2$ and $bb=4.0000$
Which 4.0000 being ſubtracted from the number given 69169, then there
Remains of the Number given 29169

Remains of the Number given,		29169
Root decuplate	$b=20$	
Divisor	$2.b$ 40.00	
The second single root	$c=6$	
$2.bc$	240.00	
cc	36.00	
	276.00	
Subtract		276.00
Remains of the Number given		1569
The Root increased	$b=26$	
Root increased and decuplate	$b=260$	
Divisor is	$2b=520$	
The third single Root	$c=3$	
$2bc$	1560	
cc	0009	
Totall	1569	
Subtract		1569
Remains of the Number given		0000

The Root increased 263, is therefore the true Root, as may be proved by recomposition, or multiplying 263 by 263, for the Product will be 69169, which was the number given.

The

The Ciphers which are put after in the Divisors and Subtracts, are only to fill up the number of places, by which the number given, or rather the remaining Points would else exceed. For the like purpose is used the decuplation of the Roots, as only to supply a place untill another figure succeed in place of the Cipher.

And in nothing else doth this work differ from the ordinary Extraction of the Square Root, commonly taught and known.

The reason of it depends upon the 4th. *Prop.* of the second Book of *Euclide*, where it is demonstrated, that *If a right line be divided by chance into two parts, the Square made of the whole, is equall to the Squares of the parts, and to the Rectangle made of the parts twice,*

So it is here as followeth.

The Square of the greater part, that is, of 260	$bb = 67600$
The Square of the lesser part, that is, of 3.	$cc = 00009$
The Rectangle of the parts, that is, 260 into 3 twice.	$2bc = 01560$
Equall to the whole Square.	69169

Nor do these letters represent so naturally the things themselves in a divided *Superficies* only, but as properly and clearly the parts of *Solid Bo-*

 dies,

dies, of which, two or three Examples for satisfaction.

In which I admonish the Reader, to be intent to the severall pointings of the quantities according to their due order, as is before expressed, and also to the placing of the Divisors and Substracts by Ciphers, as before also is intimated: for this to the Ingenious is enough, & a long verbosity to others will scarce be so.

Of Cubicall Æquations.

Let there be a Cube $aaa = fff$

Or proposed in Numbers $aaa = 41781923$

Put (as before) $b + c = a$

Then their Cubes also shall be equall.

That is $bbb + 3bbc + 3bcc + ccc = 41781923$

The Resolution.

The Homogeneall Number given $\dot{4}17\dot{8}19\dot{2}\dot{3}$

The first single Cubique Root $b = 3$

And $bbb = 27.000000$

Subtract 27.00000

Remains of the Number given $\dot{1}478\dot{1}92\dot{3}$

Re-

Remains of the Numbergiven 14781923

The first Root decuplated } $b = 30$

$3.bb$ 2700.000
$3.b$ 0090.000

Divisor 2790.000
Second single Root $c = 4$

$3.bbc$ 10800.000
$3.bcc$ 01440.000
ccc 00064.000

12304.000

Substract 12304.000

Remains of the Number given 02477923
The Root increased $b = 34$

Root increased decuplate } $b = 340$

$3.bb$ 346800
$3.b$ 001020

Divisor 347820
The third single Root $c = 7$

$3.bbc$ 2427600
$3.bcc$ 0049980
ccc 0000343

2477923

Subtract 2477923
Remains lastly of the number given 000
The Root increased $b + c = 347$

Which is the true Root of the Cube 41781923, as may be proved by recompoſition, that is, by Multiplying 347 by 347, and the Product again by 347, the laſt Product ſhall be equall to the Cube which was given to be reſolved.

And as above in the Square the Canon of the Reſolutions was the letters $bb + 2bc + cc$, being the true Square of $b + c$. And thoſe letters did anſwer exactly to the parts of the Square divided alike in both the Dimenſions: So here alſo the Canon of Reſolution, or the letters $bbb + 3bbc + 3bcc + ccc$, do exactly anſwer to the Parts or Members of a Cube, divided into two parts, alike in all the three Dimenſions, as any one may prove upon a Cube made of ſome ſlender matter, and cut through all three wayes, for he ſhall finde the whole Cube (ſuppoſed equal to 41781923 as before) juſtly made up of the two Cubes of the two ſegments, that is, bbb and ccc, and three Parallelepipedons, whoſe length and breadth are equall to b, and their thickneſſe to c, thoſe three are the $3bbc$. And laſtly, three other Parallelepipedons, whoſe length and breadth are equal to c, and their thickneſſe to b, ſuch are the $3bcc$.

See the following Schematiſme.

Th:

The Cuhe of the greater Segment which is 340, *bbb*	39304000
The three greater Parallelepipedons, 3*bbc*	2427600
The three leſſer Parallelepipedons, or 3*bcc*	46980
The Cube of the leſſer Segment, which is 7, *ccc*	343
The whole Cube given	41781923

Note, That the greater Segment is the aggregate of all the ſingle Roots except the laſt, being duly valued by a Cipher, as here it is 340, but the leſſer ſegment is the laſt ſingle Root only, as here 7,

I have done this to let the Reader ſee, that he may be ſure let the quantity to be reſolved be great or little whatſoever, if he be carefull to make his Canon right, the letters themſelves will direct him how to frame his Diviſors and Subtracts in order to the finall reſolution, eſpecially in theſe unmixed Quantities, where the points limit how far the ſubtract ſhall advance at every operation, beginning firſt at the point next the left hand, not further, and to the ſecond point only at the ſecond work, and not otherwiſe in all that follow.

And

And in Mixed Æquations, if they be made up of Cube with addition of certain Squares, or certain Roots, or both Squares and Roots, or by Subtraction of the same, the Canon of the Resolution must ever be made by multiplying the assumed Root $b+c$ in the place of the quesititious Root a, quite through the Æquation in all the degrees thereof, for so shall arise all the severall parcels of which the severall Subtracts are orderly to be made.

In a Cubique Æquation, if all the quantities be present, there is no need to point any but the Cubiques and Roots: yet I have here distinguished the places of the Squares also with litle Crosses obliquely; which labour, when the Workman is intent upon his businesse, may well enough be spared.

Of the resolution of Mixed Cubiques.

Let the Æquation $aaa + daa - ffa = ggg$ be proposed in Numbers.

As let it be $aaa + 32aa - 75a = 29282970$

Therefore $d = 32$ and $ff = 75$

And $ggg = 29282970$

Put $b + c = a$

And

And make the Canon of Resolution by substituting $b + c$ in the place of a quite through the severall quantities $aaa + daa - ffa$. The Canon rightly made will be $+ bbb$

$$
\begin{array}{llll}
+ bbb & & & \\
+ 3bbc & + dbb & - ffb \\
+ 3bcc & + 2dbc & - ffc \\
+ .ccc & + dcc & \\
\end{array}
$$

These severall parcels of the Canon, being rightly subtracted from the homogeneall Number 29282970, the Number shall be thereby resolved, and the Root a found.

☞ Note first, That all the parcels in the Canon, which have not the secondary Root c in them, as $+ bbb + dbb$ and $- ffb$, are to be subtracted at the first Operation, the other remaining parcels to be all subtracted as often as there shall be points left above.

The Resolution.

The homogeneal Number given	29282970
The first single Root $b = 2$	
$+ bbb$ 8.000000	
$+ dbb$ 128.0000	
$+ 928.0000$	
$- ffb$ ..150.00	
$+ 92650.00$	
Subtract	92650.00
Remains of the Number given	20017970

Remains of the Number given	20017970

The first Root decuplate $b = 20$

$3bb$	1200.000
$3b$	..60.000
$2db$	.1280.00
d	...32.00
	13912.00
$-ff$	....75.0

Divisor 1390450

The second single Root $c = 9$

$3bbc$	10800.000
$3bcc$	.4860.000
ccc	..729.000
$2dbc$	.11520.00
dcc	..2592.00
	178002.00
$-ffc$	....675.0
	17793450

Subtract	17793450
Remains of the Number given	02224520

The Root increased $b = 29$

Root increased decuplate $b = 290$

Re-

Remains of the Number given 02224520

$5bb$ 252300

$3b$ ...870

$2db$ .18560

$-ff$ 75

Divisor 271655

Third single Root $c = 8$

$3bbc$ .2018400

$3bcc$ ...55680

ccc 512

$2dbc$ ..148480

dcc 2048

2225120

$-ffc$ 600

2224520

Subtract lastly 2224520

Remains of the Number given 000

Whereby it appears that the whole Root 298 is the true Root whereby this Æquation is explicable, as may be proved also by recomposition.

For $bbb =$ 24389000

$3bbc =$ .2018400

$3bcc =$ ...55680

$ccc =$ 512

$dbb =$ .2691200

$2dbc =$ ..148480

$dcc =$ 2048

In all = 29305320

Frō which subtract $ffb + ffc =$ — ...22350

Remains 29282970

Which was the whole Homogeneall Number given.

NOTE.

Whereas in composing the Divisor all the gradual quantities are used, as in the former example, $3b$ and d, aswell as $3bb$ and $2db$, it is to be noted that in practice, those smaller particles $3b$, &c. May be omitted; the other without them ministring light enough for choosing the Secondary Roots.

Having now instanced in an Example where all the powers were present, in these one or two that follow, to make the work shorter, I shall leave out one or other of them.

In the Æquation $aaa + ffa = ggg$.

Propounded in Numbers $aaa + 320406a = 8348132$, It sometimes happens that the Coefficient abounds with more binarie figures then the homogeneal doth with ternaries, in such a case that there may be rooom made to begin the Extraction. The Coefficient must be devolved to the next point further to the right hand, or to the second, third, fourth, or further, if need require, and there the work is to begin. The Coefficient is alwayes the known quantity which multiplies any of the unknown inferior quantities.

Ex-

Example of Devolution.

$$aaa + 320406.a = 8348132$$

Put $b + c = a$

The Canon will be $\left\{\begin{array}{l} bbb + 3bbc \\ + 3bcc + ccc \\ + ffb + ffc \end{array}\right\} = 8348132$

Resolution.

The Homogeneall number given		8348132
The first single Root $b = 2$		
	$+ bbb$ 0008.000	
	$+ ffb$ 640812.0	
	$= 6416.12.0$	
Subtract		641612.0
Remains of the Number given		1932012
The first Root decuplate $b = 20$		
	$3bb$ 00001200	
	ff 00320406	
Divisor	321606	
The second single Root $c = 6$		

Remains of the number given 1932012

$$\begin{array}{ll} 3bbc & \ldots\ldots 7200 \\ 3bcc & \ldots\ldots 2160 \\ ccc & \ldots\ldots 216 \\ ffc & .\,1922436 \\ \hline & 1932012 \end{array}$$

Subtract 1932612

Remains of the number given 000

Wherefore the whole Root is equall to the Root increased, 26, as may be proved in manner as beforesaid.

It sometimes happens also in the Æquation $aaa - ffa = ggg$ Put into Numbers.

As $aaa - 105000.a = 203125$.

That the Coefficient abounds with more binarie figures then the homogeneall with ternaries: Wherefore that there may be place for the Resolution, put before the homogeneall, toward the left hand, so many Ciphers as will afford that to receive as many Cubicall points, as the Coefficient doth Quadraticall: And at the first empty point, as it were by anticipation, begin the Resolution. In which there is this of Compendium, that the first Square Root extracted out of the Coefficient, is either equall to the first single Root

Root of the homogeneall ſought, or leſſe then it by Unity.

But if the Æquation had but two Dimenſions, As $aa - 254a = 65024$, then the firſt figure of the Coefficient, namely 2, is the firſt Root.

Example of Anticipation.

The homogeneall Number given $+\dot{0}20\dot{3}12\dot{5}$

$b + c = a$

The Canon is $\begin{cases} bbb + 3bbc + 3bcc + ccc \\ \quad - ffb - ffc \end{cases}$

The Reſolution.

The firſt ſingle Root $b = 3$

$+bbb$	27.000000
$-ffb$	3150.0000

Subtract the differẽce which is $\Big\}$ -45000.00

Remains of the Number given $+47\dot{0}3\dot{1}2\dot{5}$

The firſt Root decuplate $b = 30$

$+3bb$	2700.000
$-ff$	105000.0
Diviſor	165000.0
$3bbc$	5400.000
$3bcc$	.360.000
ccc	...8.000
$-ffc$	210000.0
	$+366800.0$
Subtract	$+366800.0$

Remains of the Number given $+1035125$

Remains of the Number given		$+1035125$
The Root increased $b = 32$		
Root increased decuplate $b = 320$		
$3bb$	307200	
$-ff$	105000	
Divisor	202200	
The third single Root $c = 5$		
$3bbc$	1536000	
$3bcc$	$..24000$	
ccc	$....125$	
$-ffc$	$.525000$	
	1035125	
Subtract		1035125
Remains of the number given		000

Which sheweth that the Root increased, $b + c = 325$, is the true Root of the Æquation, And it may be proved by recomposition as formerly.

In the Æquation $-aaa + ffa = ggg$, Which is explicable by two Roots, as shall be shewed in the next Chapter, *Section* 5, to finde them both. Put the Æquation into Numbers.

As

As $-aaa+52416a=1244160$

Therefore $ff=52416$ & $1244160=ggg$

Put $b+c=a$

Therefore $\left.\begin{array}{l}-bbb+ffb\\-3bbc\\-3bcc+ffc\\-ccc\end{array}\right\}=1244160$

Extraction of the greater Root.

The Homogeneall Number given $\dot{1}24\dot{4}16\dot{0}$

The first single Root $b=2$

$$\begin{array}{lr} ffb & 104832.00 \\ -bbb & 8.000000 \\ \hline & +24832.00 \end{array}$$

Subtract $+24832.00$

Remains of the Number given -1239040

The first Root decuplate $b=20$

$$\begin{array}{lr} ff & 52416.0 \\ -3bb & 1200.000 \\ \hline \end{array}$$

Divisor -67584.0

The second single Root $c=1$

Remains of the number given		-1239040
ffc	$.52416.0$	
$-3bbc$	1200.000	
$-3bcc$	$..60.000$	
$-ccc$	$...1.000$	
	-736840	
Subtract		-736840
Remains of the number given		-502200

The Root increased and decupled $b = 210$

ff	52416
$-3bb$	132300
Divisor	-79884

The third single Root $c = 6$

$+ffc$	314496	
$-3bbc$	793800	
$-3bcc$	$.22680$	
$-ccc$	$...216$	
	-502200	
Subtract		-502200
Remains of the number given		000

Root increased $b + c = 216$, which is the true Root sought.

2. Edu-

2. Eduction of the lesser Root by Devolution.

The homogeneall Number given 1244160

$b = 2$

ffb 104832.0

$-bbb$ $-$ ...8.000

$+$ 1040320

Subtract $+$ 1040320

Remains of the Number given $+$ 203840

The Root increased and decupled $b = 20$

ff .524.16

$-3bb$ $+$.1200

Divisor .51216

The second single Root $c = 4$

ffc 209664

$-3bbc$..4800

$+3bcc$...960

$-ccc$....64

$+$203840

Subtract $+$203840

Remains of the Number given 000

The Root increased $b + c = 24$

Wherefore 24 is the true Root ſought, as may be proved by recompoſition, as hath been ſhewed before.

So this Æquation is explicable by two Roots, that is, 216, and 24.

VIETA, *Lib. de Recognitione æqnationum*, *Cap.* 18. *Prop.* 2. ſaith, That in the Æquation $-aaa+ffa=ggg$, the Coefficient ff is compoſed of three proportional Squares, and the Homogeneall ggg is made by Multiplication of the aggregate of the two firſt, or the two laſt, (for all is one) into the ſide of the other, and the Root a may be the ſide either of the firſt or third. This (or the ſame in ſubſtance) ſaith that Noble Author, And it is evident, for make

$$cc'+dd''+hh'''=ff$$

And pnt $c=a$

Therfore $ccc+ddc+hhc-ccc=ddc+hhc$

Or put $h=a$

It is $hhh+ddh+cch-hhh=ddh+cch$

Both which are manifeſt

COMPENDIUM 1.

Hence it may be ſhewed, that either of the quæſititious Roots, as a, being found and called c, the other Root e may be found by a Quadratique

tique Æquation only. For ſuppoſing

$$ee + ce = ff - cc,\ \text{Then}$$

$$\text{It is } ee + ce + cc = ff.$$

And $cc' \quad ce'' \quad ee'''$ *Euclide* 6. 23.
But by conſtruction $cc' \quad dd'' \quad bb'''$. And $cc + dd + bb = ff$. So then $bb = ee$ and $b = e$.

But it was ſhewed before that b might be a Root of this Æquation $- aaa + ffa = ggg$ And therefore e alſo is a Root of the ſame, and the Compendium is proved.

Example alſo in Numbers.

In the laſt Æquation $aa = cc = 46656$

And $\quad ff = 52416$
From which take $\quad cc = 46656$

$$\overline{05760}$$

Remains $\quad ff - cc = 5760$
But $\quad ee = 576$
And $\quad ce = 5184$

$$\overline{5760}$$

The Summe is $\quad ee + ce = 5760$
Therfore $ee + ce = ff - cc$, Which was, &c.

In the Æquation $- aaa + faa = ggg$,

the Coefficient f is composed of three proportionall lines, and ggg is equall to a Solid made by a Square (whose side is equall to the two first, or the two last) multiplyed into the remaining line: And the aggregate of the first and second may be a, and the aggregate of the second and third shall be e. Put $1'$ $2''$ $4'''$

And suppose $-aaa+7aa=36$

Then a may be 3, and e is 6. *Vieta, de Recognit. Cap.* 18. *Prop.* 6.

COMPENDIUM 2.

And therefore the Root a found, and called c, the Root e may be found by a plain Æquation; for suppose the middle proportionall y, it is $f-y-c'$ y'' c'''.

And $fc-cy-cc=yy$ Or, $yy+cy=$ $=fc-cc$. And making $fc-cc=xx$, it is $yy+cy=xx$. And the Root y being found by the first Rule of *Chap.* 2, It is lastly (making c' y'' d''') $y+d=e$.

I will here adde a few Rules (grounded upon Mr. *Harriots* 6 Sections) by which the Reader may easily perceive the Fabrique of Æquations, their Roots, increment and decrement, Multiplication and Division of them, and their number in any Æquation as followeth.

CHAP. IV.

Rule 1.

Every Æquation being composed of some known and some unknown quantities hath its Originall by roots composed of a quantity known and of one other quantity unknown, and these roots multiplied together produce certain particular Members with $+$ *and* $-$ *respectively signed (for in every æquation both these signes are present) which orderly placed make up the æquation.* As the æquation $aa - ba - ca + bc = 0.$ is made by multiplying $a - b = 0$ by $a - c = 0.$ And because it was at first $a - b = 0$ therefore $a = b$ and the like of $c.$ And from hence it follows that where the first terme (or highest power) in a quadratique Æquation is signed $-$ there the Æquation hath two roots, as here by substracting on both parts $+ aa - ba - ca,$ the Æquation will be $bc = - aa + ba + ca,$ and must have 2 roots.

1 These compound quantities so multiplying I shall call *Binomialls*, whether $a + b$ or $a - b.$ not having any need in this treatise to distinguish betwixt *Binomials* and *Residuals*.

2 The æquation $aa - ba + ca = bc.$ If it be, $b < c.$ put $c - b = d.$ then the

the æquation will be, $+aa+da=bc$, and is of the first kind mentioned in *Chap.* 2. but if it be $b>c$, put $b-c=f$ and the æquation will be $+aa-fa=bc$, and is like the second sort in the same Chapter.

The Originall of the æquation $aa-ba+ca-bc=0$ here proposed, is $+a-b=0$ multiplied by $+a+c=0$, that is $a=b$ by $a=-c$. This æquation hath but one true root, which is b, and one false, which is c.

3 By this which hath been said it is plain that some æquations have as many roots as dimensions, some not so many, but none can have more; for the number of dimensions being the same with the number of multipliers (if all diverse) can be but all roots. Nor can the æquation be divided by any other thing then one of those Binomials by whose multiplication it was made.

But if the multipliers how many soever be still the same, there can be but one root. For let $+a-b=0$ be muliplied *Biquadratically*, the product is $+aaaa-4baaa+6bbaa-4bbba+bbbb$. where it is plain there can be no other root but b. I mean none greater or lesse then it: because in truth here are 4 roots, but every one singularly equall to b.

For if there may, let it be d, and let d be greater or lesse then b, it imports not which. And seeing $d=a$, Substitute d in the place of a, quite through the Æquation, it will be

$dddd$

$dddd - 4bddd + 6bbdd - 4bbbd + bbbb = 0$. Which if $d > b$, or elſe $d < b$, is at firſt ſight impoſſible: For the difference between the $+$ and $-$ is alwayes equall to the power of the difference between b and d, which power is here a Biquadrat, therefore $d = b$; And again ſeeing this Æquation may be derived by putting b equall to d, for ſubſtituting b in the place of d quite through, It will be

$$+2bbbb + 6bbbb = 4bbbb + 4bbbb$$

Which is manifeſt, therefore again $b = d$, which is contrary to the ſuppoſition, therefore b is the only Root of this æquation, for indeed, the æquation propoſed being made only of multiplications of $a - b = 0$ cannot be divided, that is reſolved, by any other Binomiall then $a - b$, of which it was made,

4 Hence it is that the laſt term in every æquation may be called the Homogeneall, becauſe it is naturally made by multiplication of the Roots of the æquation, though the coefficients in ſome ordinary æquations are diſguiſed with other characters, which happens by Addition or Subtraction of them, to reduce the canonicall æquation to fewer members, whereby the redundancie of the ſignes $+$ and $-$ is to be taken away, this is to be ſeen above in this Rule, where the æquation $+aa - ba + ca - bc = 0$ is reduced to

$+aa$

$+aa+da-bc=0$, by making $d=c-b$ and $+aa+ba+ca-bc=0$, reduced to $+aa-fa-bc=0$, by making $b-c=f$ Where the Coefficient d or f, is not a part of the Homogeneall bc, but a difference by which b is greater or lesse then c: by help of which difference, the æquation which consisted canonically of four Members, hath now but three.

5 And this Reduction is usefull, for as Mr. *Des Cartes* saith, and which may be seen true by the way of Multiplication above shewed, every æquation hath so many true Roots as the Signes $+$ and $-$ therein are changed, which in the canonicall æquation

$+aa-ba+ca-bc=0$, are changed three times, whereas the æquation hath not three true Roots, but one true and one false, that is b & c, and the common æquation reduced changeth the signes but once, that is, from $+da$ to $-bc$ in the former; or from $+aa$ to $-fa$ in the later: and from thence it may be known that the æquation hath but one true Root. The like consideration ought to be in others.

And whereas the said *Des Cartes* doth often mention false Roots, it is to be noted that such are lesse then nothing, as $+a+b=0$: Or $+a=-b$, & if any true root, as $+a-c=0$ be multiplyed by this $+a+b=0$, there will arise an æquatiō $+aa+ba-ca-bc=0$

where

where the ſigne $+$ follows twice, the ſigne $-$ twice, and they are once changed, which ſhould intimate (according to *Des Cartes*) two falſe Roots, and one true: for he ſaith, So many times as $+$ or $-$ come twice together, ſo many falſe roots there are, this æquation therefore muſt be reduced, by making $b - c = d$ if $b > c$, or elſe if $b < c$ then make $c - b = f$, ſo it will be either $+ aa + da - bc = 0$, Or $+ aa - - fa - bc = 0$ which confirms that which *Des Cartes* ſaith of twice $+$ or $-$: Namely, that there are as many falſe roots in the æquation, as $+$ or $-$ come twice together, and ſo many true roots as $+$ and $-$ are changed.

And where the Roots are all falſe, the æquation is impoſſible, as $a + b = 0$ multiplyed by $a + c = 0$, produceth $aa + ba + ca + bc = 0$ which cannot be. And therefore when there is an æquation pretended like $aa + ba + ca = = - bc$, preſent judgement may be made.

6 The ſame *Des Cartes* ſaith alſo that all the falſe Roots in any æquation, may be turned to true ones, and the true ones to falſe, by changeing the ſignes of the ſecond, fourth, and every even term. And this is evident, for of the æquation $a^4 - 2a^3 + 10aa - 30a - 87 = 0$ by ſuch change is made $+ a^4 + 2a^3 + 10aa + 30a - 87 = 0$ where the firſt had three true Roots, and but one falſe, the later hath three falſe and but one true. This Æquation was taken

ken at all adventures, to ſerve for an Example only, whereas any other whatſoever will doe the like.

Rule 2.

The unknown roots of an Æquation may be increaſed or decreaſed, by ſuppoſing another unknown quantity + or — the decrement or increment, and of that Binomiall compoſing the æquation as it was before of the firſt unknown quantity: and if this increment be put equal to ſuch a part of the coefficient of the ſecond terme, as unity is of the dimenſions of the firſt terme (if the ſignes of the firſt and ſecond be both + or both —) or if the Decrement be made equal to ſuch a part of the ſaid coefficient as unity is of the dimenſions as aforeſaid, (if the ſignes of the firſt and ſecond terme be one + the other —) then by ſuch increaſe or decreaſe of the root the ſecond terme of the æquation ſhall be taken away, and anulled.

Example.

In the æquatiō $+aaa+baa-bbc=0$, the Roote a may be increaſed by making $e-q=a$, and ſubſtituting $e-q$ in the place of a quite through the æquation, and thereby ſhall ariſe a new æquation:

$$\left.\begin{array}{r} +eee-3qee+3qqe-qqq \\ +bee-2bqe+bqq \\ -bbc \end{array}\right\}\begin{array}{l} =+aaa \\ =+baa \\ =-bbc \end{array}$$

Which

Which is equall to the former as you see agreeing in the particulars, and the root *e* being found, *a* may be had by casting away *q* from *e*.

And because the number of the dimensions of the first terme *a a a* is 3, if according to the later part of the rule the quantity *q* be proportioned, by making $3'\ 1''\ b'\ q''$ then $b = 3q$ and $+bee$ will destroy $-3qee$, and so the second term *e e* wil be quite taken out of the æquation, as is manifest, for the æquation so purged will be $+eee - 3qqe + 2qqq - bbc = 0$ And by subtracting on each patt $+2qqq - bbc$ having first made $bbc - 2qqq = ddd$, it will be then $+eee - 3qqe = ddd$. The manner of such reduction of Solids, shall follow in the next Chapter.

In like sort the Root *a*, might have been decreased by any quantity, as *x*, which if it be proportioned to *b* as aforesaid, would take away the second term of an æquation, where the signes of the first and second terms are not like; as in the æquation $+aaa - baa - bbe = 0$, by putting $3x = b$, and $e + x = a$: the Probleme will be fully performed by making $e + x$ the Root of the new æquation, as before was $e - q$, observing the same order in composing the particulars, due respect had to the signes + and —, where they ought to be altered.

The

The former reduced æquation $+e^3 - 3qqe = ddd$ might be further reduced (if need require) to $+e^3 - bqe = ddd$.

NOTE.

This augmentation and diminution of the Roots in ſuch manner as to take away the ſecond term of any æquation, is of excellent uſe in ſuch æquations as have three or four dimenſions, and cannot by any diviſion with any binomiall made of $a +$ or $-$ ſome other known quantity, as b, c, or the like, be reduced to fewer dimenſions, whereby it is certain that ſuch an æquation is Solid, and cannot by any artifice already, or likely to be invented, be reſolved by Ruler and Compaſs, but by any of the Conique Sections it may; in this caſe it is either neceſſary or extreamly facilitating, to take away the ſecond terme (if there be any) from the æquation, as ſhall be ſeen hereafter in its place.

Rule 3.

The unknown Root of any Æquation may be multiplyed (or divided) by any known quantity multiplying (or dividing) the ſecond term of the Æquation by the ſaid quantity, the third by the Square, the fourth by the Cube thereof, and ſo forward continually in this order, as often as there are terms in it, having firſt aſſumed another unknown quantity, ſo multiplex to the ſaid unknown Root, as is required. Ex-

Example.

In the Cubicall *Æquation* $a^3 + baa + cca - bcd = 0$: Let it be required to multiply the Root a by 4.

Assume $e = 4a$ and write

$$eee + 4bee + 16cce - 64bcd = 0$$

Which is an æquation, and the root e is quadruple to a, as may be proved thus.

Put $a = 4$ $b = 3$ $c = 2$ and $d = 21\frac{1}{3}$

$$\text{Then}\quad \left.\begin{array}{r} aaa = 64 \\ baa = 48 \\ cca = 16 \\ \hline \text{In all}\quad 128 \end{array}\right\} \text{But } bcd = 128$$

Therefore $a^3 + baa + cca - bcd = 0$

Again, Put $e = 16$ All else the same still.

$$\text{Then}\quad \left.\begin{array}{r} eee = 4096 \\ 4bee = 3072 \\ 16cce = 1024 \\ \hline \text{In all}\quad 8192 \end{array}\right\} \text{But } 64bcd = 8192$$

Therfore $e^3 + 4bee + 16cce - 64bcd = 0$

And $e = 16 = 4a$ Which was to be proved.

The utility of this Rule wil appear in reducing æqua-

æquations affected with fractions, to whole numbers, by multiplying the Roots by the denominator or denominators of the fraction, for by ſuch means the coefficient of the ſecond term is multiplyed by the ſame as before, multiplying a by 4, multiplyed alſo b by the ſame number 4. And many times by this Rule æquations may be freed from Surd Numbers alſo; eſpecially if ſuch be found in the ſecond term, as is eaſie to be ſeen by triall, for if there be an æquation ſo affected,

$$\text{As } aaa + \sqrt{8}\,aa + \tfrac{29}{24}a - 4\sqrt{2} = 0$$

$$\text{Put } e - \sqrt{8} = a$$

$$\text{And write } + eee + 8ee + 9\tfrac{2}{3}e - 128 = 0$$

So the Surds are vaniſhed.

But if yet it be required to avoid the Fraction $9\frac{2}{3}e$, then make $y = 3e$. And multiplying 8 by 3, $9\frac{2}{3}$ by 9, and 128 by 27, there wil be a new third æquation.

$$+ yyy + 24yy + 87y - 3456 = 0$$

Which conſiſts of entire Numbers, having one true Root which is 9; and the Root of the middle æquation was 3, which is the third thereof; and the Root of the firſt æquation was $3 - \sqrt{8}$ And now I hope this Rule and the uſe of it is plain enough.

NOTE I.

It may be noted, that if the Surds in the ſecond and

and last termes of the first æquation, to wit, $aaa + \sqrt{8aa} + \frac{29}{24}a = 4\sqrt{2}$ had been utterly incommensurable, the reduction had not been so fesible. For although $4\sqrt{2}$ multiplied by the cube of $\sqrt{8}$ that is by $8\sqrt{8}$ produceth $32\sqrt{16}$. which is equall to the intire number 128, yet if it had been $2\sqrt{3}$ or $2\sqrt{5}$, or any such primes to be multiplied by $8\sqrt{8}$ the product would have been $16\sqrt{24}$ or $16\sqrt{40}$. though this last may (by the note after the Consectarie in *Chap.* 6.) be reduced by multiplying it again by $\sqrt{40}$ unto the intire number 640. Neverthelesse this second multiplication by a *Surd*, renders the æquation inexplicable, at least by the precedent Rule.

NOTE 2.

It may be further noted, that if instead of $e = 4a$ one would put $e = fa$ lines not being so liquid as numbers, the æquation would then be $eee - fbee + ffcce - fffbcd = 0$ increasing the dimensions of the lesser termes, for remedy whereof three lines are to be found in proportion one to another as are the magnitudes $fb . ffc . fffb$. of which let the first line be supposed to containe *Unity* as often as the superficies fb doth (for which purpose *Unity* must be a line set, and agreed on before.) The names of

 these

these lines when found may be called g, h, k, and the æquation may be written

$$+eee+gee+hce-kbc=0.$$

NOTE. 3

But it is againe to be noted, that where the lines f, b, and c, are commensurable in length the three lines k, h, g, may be very easily found, for then they may be signified by numbers and if f be put for *Unity* then $e=a$ and the work frustrate, but where the said lines are incommensurable in length this Reduction is alwayes hard if not impossible: For those incommensurable lines doe most commonly represent such surd numbers as cannot by any Reduction be compared.

Rule 4.

The Æquation $aaa-3bba=2ccc$; *or any other like it, by putting* $\frac{ee+bb}{e}=a$ *may if* $c>b$ *be brought to* $eee=ccc+ddd$ *or if* $c=b$ *to* $eee=ccc$, *or lastly, if* $c<b$ *then to* $eee=ccc+\sqrt{-dddddd}$: *Which last may be called an impossible Æquation.*

Put] e' b'' $\frac{bb'''}{e}$ And because a is equall

to the ſum of the Extreams, which are $e + \frac{bb}{e}$ therefore,

From thence it will be

$$\left.\begin{array}{l} \dfrac{+e^6 + 3bbe^4 + b^4ee + b^6}{eee} = +aaa \\[2ex] \text{And } \dfrac{-3bbe^4 - 3bbbbee}{eee} = -3bba \end{array}\right\} = +2ccc$$

Therefore rejecting the contradictories, and multiplying all by eee, it is, $+e^6 + b^6 = = 2c^3e^3$.

Therefore $+e^6 - 2cccce^3 = -bbbbbb$.

And $+e^6 - 2c^3e^3 + c^6 = + c^6 - b^6$.

Therefore, (for $e^3 - c^3 = \sqrt{e^6 - 2e^3c^3 + c^6}$)

$+e^3 = ccc + \sqrt{c^6 - b^6}$.

If now in the firſt caſe c be greater then b, then put $c^6 - b^6 = d^6$.

Then it will be $eee = ccc + \sqrt{dddddd}$

That is, $eee = ccc + ddd$. Which is the Æquation promiſed in the firſt caſe.

Secondly, If b be equal to c, then $c^6 - b^6 = 0$ And it will eaſily follow, ſeeing (as is ſhewed a-

 bove)

bove) that $e^6 - 2c^3e^3 + c^6 = 0$, therfore the Root of it $e^3 - c^3 = 0$, that is $eee - ccc$ the second æquation prescribed.

Lastly, by the third case, seeing c is lesse then b, Put $cccccc - bbbbbb = - dddddd$: Then it will be $eee = ccc + \sqrt{-dddddd}$ the equation prescribed in the third case, and (because of the inexplicability of $\sqrt{-dddddd}$) impossible.

COMPENDIUN

Whereas Mr. *Harriot* saith *Propter* $\sqrt{-d^6}$ *inexplicabilitem*, *&c.* The said quantity $\sqrt{-d^6}$ is not explicable because $-d^6$ ariseth by multiplying $+d^3$ by $-d^3$ betwixt which two there is no meane; for no one thing can produce d^6 but d^3 onely, and $-d^6$ is not produced by $+d^3$ or $-d^3$ because by both, this therefore may serve for a *Compendium* to save labour which might else be lost, in seeking that which is impossible to be found.

NOTE.

I use b^6 for $bbbbbb$, and b^4 for $bbbb$, and b^3c^3 for $bbbccc$, and the like, (as *Des Cartes* hath done) only for abridgement, as in the Definitions of the Powers is already shewed.

And

And $ccc + \sqrt{c^6 - b^6}$ with that line over to diſtinguiſh betwixt $\sqrt{c^6 - b^6}$ as one quantity, and $\sqrt{c^6}$ taken by it ſelfe and $-b^6$ taken apart alſo, for by ſuch miſtakes may great errors ſucceed.

I will adde no more rules, theſe 4 may be multiplied by any one that doth not find theſe ſufficient for his purpoſe, at his own pleaſure.

CHAP. V.

Of Reduction of Solids.

Having ſpoken in *Chap.* 4. *Rule* 2. of making $bbc - 2qqq = ddd$, and in *Rule* 4. of $c^6 - b^6 = dddddd$, I think it not amiſs here to ſhew how ſuch Addition and Subtraction of Solids may be performed.

And it may be noted that ddd, is for brevity ſake there uſurped for ggc. or ſome other ſolinomial rectangle *Parallelepipedon.* equall to the *Binomial* rectangle Solid $bbc - 2qqq$, for if this *Binomial* could (by plaine Geometrie be) given in a *Cube*, as is ddd, ſomething elſe might be done which here I will not ſpeake of.

Now therefore ſeeing $b = 3q$ as there it is, the æquation may be written

$9qqc - 2qqq = ddd$, or rather

$9qqc - 2qqq = ggc$.

Make $\frac{qq}{c}=f$, therefore $2qqq=2qfc$,

Secondly, make $9qq-2qf=gg$, from thence it is plain that $bbc-2qqq=ggc$, which was first to be done.

Secondly, to reduce c^6-b^6 into one intire Solid, though not into a Squared Cube as d^6, as is usurped by M^r. *Harriot* for brevity in writing, or facility in reasoning, *Pag*. 100, supposing that done which cannot be done by streight lines and circles hitherto.

Now therefore seeing c^6-b^6 is produced by multiplication of $ccc+bbb$ into $ccc-bbb$.

Make $\frac{cc}{b}=f$, and $\frac{bb}{c}=g$, & $f+g=q$

and $f-g=p$, therefore $bcf=ccc$, and $bcg=bbb$, and $bcq=ccc+bbb$. Secondly also $bcp=ccc-bbb$: And therefore $bbccpq=cccccc-bbbbbb$, which was secondly to be done.

Example in Numbers.

Put $b=2$ and $c=3$: Then $ccc^3=729$, and $bbbbbb=64$, and then $cccccc-bbbbbb$, that is $729-64=665$, which is produced by multiplying $27+8$ by $27-8$, that is, 35 by 19. Now make $f=\frac{9}{2}$, and $g=\frac{4}{3}$

$g = \frac{4}{3}$, then $f + g = 5\frac{1}{6} = q$, and $f - g = 3\frac{1}{6} = p$. And $bcq = 35$, and $bcp = 19$. And laſtly $bbccpq = 665$.

Moreover, if you make $pq = xx$, the Solid is further reduced to $bbccxx$, which although it be not a Squared Cube, yet it hath a ſquare root, namely bcx, which may be of good uſe in many caſes to reſolve Æquations into Analogiſmes, of which kinde of Demonſtration, by help of *Euclide* 6. 14. ſome notice is taken before in *Chap.* 2.

NOTE.

The three caſes of the æquatiõ $a^3 - 3bba = 2c^3$, mentioned in the beginning of the fourth *Rule* of the laſt *Chap.* are called by Mr. *Harriot*, the firſt *Hyperbolicall*, the ſecond *Parabolicall*, the third *Ellipticall*, becauſe of ſome ſimilitude between them and thoſe ſections, of which three Caſes, the firſt is reſoluble by a Conique Section, the ſecond by a Circle, and the third not at all.

Multiplication and *Diviſion* of Solids is altogether as eaſie as Addition or Subtraction, for if one would divide ccc by bb, make $\frac{cc}{b} = x$, and again make $\frac{cx}{b} = z$, then z is the Quotient required.

Ex-

Example in Numbers.

Put $b = 2$ and $c = 3$, then $\frac{ccc}{bb} = 6\frac{3}{4}$,

to finde which, make $\frac{cc}{b} = x = \frac{9}{2}$, then

$cx = \frac{27}{2}$, & $\frac{cx}{b} = \frac{27}{4} = z = \frac{ccc}{bb} = 6\frac{3}{4}$,

as it ſhould be.

Again, if c^5 ſhould be divided by b, it is now

$\frac{ccc}{bb} = z$, and multiplying by b it is $\frac{ccc}{b} = bz$:

Again, multiplying by cc it is $\frac{c^5}{b} = bccz$,

and $bccz$ is the Quotient required.

But if it be required to bring the quotient to a Biquadrat, make $bz = dd$, then $ccdd = bccz$. And make $cd = ff$, then the quotient will be $ffff$.

Multiplication is naturally ſo eaſie that there needs no more be ſaid of it, then what hath been ſaid already in *Chap.* 1.

Now, of æquations consiſting of 3 termes in continuall proportion as $a^4 + bbaa = c^4$ or ſecondly $a^6 - bbba^3 = c^6$, or laſtly let it be $-a^8 + bbbba^4 = c^8$, let them firſt be propoſed in numbers as $a^4 + 2aa = 24$, if by

Rule

Rule 1 of *Chap.* 2. it be wrought, it will be found $\surd\ 25 - 1 = aa$, and $aa = 4$ or $a = 2$.

Otherwise if the square of halfe the coefficient be added on both parts, then
$a^4 + bbaa + 1 = 25$.

And their square roots also are equall; that is $aa + 1 = 5$ and $aa = 4$ or $a = 2$ as before, and the latter may prove the former.

2 In the second, let it be $a^6 - 10aaa = 459$ Adde 25 to each part, then it it is
$aaaaaa - 10aaa + 25 = 484$.

Now each part of the æquation is a Square & their Roots also are equal; that is $aaa - 5 = 22$, that is $aaa = 27$, and $a = 3$.

3 Lastly, If $-a^8 + 700a^4 = 46875$ from the Square of $\frac{700}{2}$, that is, from 122500 take the homogeneall 46875, there remains 75625, whose square root is 275. And either $350 + 275$. Or $350 - 275$, that is either 625 or 75 is equall to $aaaa$, and $a = 5$. Or $\surd qq.\ 75 = a$, which Charracter $\surd qq.$ signifies the Biquadraticall Root.

NOTE

The first and last of these three æquations, may be done aswell in Lines as Numbers (by the said three Rules of *Chap.* 2. and so any æquation of 4, 8, 16, or. 32 dimensions, but æquations of 6, 12, or 24 dimensions, cannot be effected so,

be-

because there is ever one or more Cubique roots to be extracted, which without two meanes cannot be done.

For if it may, then I say, that two meanes between any two lines may thereby be found, for in the second æquation $aaaaaa - bbbaaa = c^6$ by Rule 2 *Chap.* 2 $c^6 + \frac{1}{4}b^6$ is a square, make $\frac{cc}{b} = d$, then $bd = cc$, and $bbdd = c^4$, and $bbccdd = cccccc$, then make $\frac{1}{4}\frac{bb}{c} = f$ therefore $fc = \frac{1}{4}bb$, and $fcb^4 = \frac{1}{4}b^6$. Now because $\frac{bb}{c} = 4f$, make $h = 4f$, then $fcchh = \frac{1}{4}b^6$.

Make $fc = ll$, then $cchhll = \frac{1}{4}b^6$. Again, make $bb + hh = mm$, and $dd + ll = nn$. And then it will be . . $ccmmnn = bbccdd + cchhll$, that is, $c^6 + \frac{1}{4}b^6$, to the square root hereof cmn, add $\frac{1}{2}bbb$: thus, make $\frac{1}{2}\frac{bb}{m} = p$; then $mp = \frac{1}{2}bb$, and $bmp = \frac{1}{2}bbb$. Make $\frac{bp}{n} = q$, then $mnq = \frac{1}{2}bbb$. Lastly, make $c + q = x$, then it is $cmn + \frac{1}{2}bbb = mnx = aaa$, by *Chap.* 2. *Rule* 2. Now if m, n, and

x

x be proportional, then the middlemoſt is equal to a, but that is uncertaine, and cannot be made otherwiſe: But by making $rr = mn$ it will be $rrx = a^3$ and a will then be the leſſer of two meanes between r and x if $r < x$ or the greater meane, if $r > x$. And ſo if r and x had been given, and required to find 2 meanes between them by *retrogradation* orderly, one might come to the ſaid æquation $a^6 - bbbaaa = c^6$ of which if the root a be found, two meanes are alſo found between r and x which was to be proved.

CHAP. VI.

Of Surd Numbers

Rule 1

THe ſquare root of any nnmber being multiplied by that number, produceth the ſquare root of the Cube of the number.

For $\sqrt{a}$ multiplied by a produceth $a\sqrt{a}$, but $a\sqrt{a} = \sqrt{aaa}$ for taking *Equimultiplices* they will be equal, as if the firſt, namely $a\sqrt{a}$ be muliplyed ſtill by $\sqrt{a}$, the product is $a\sqrt{aa}$, that is aa. And if $\sqrt{aaa}$ be multiplied by $\sqrt{a}$ it produceth $\sqrt{aaaa}$ that is aa alſo, wherefore $a\sqrt{a} = \sqrt{aaa}$. And therefore $3\sqrt{3} = \sqrt{27}$ either of which is the cube of $\sqrt{3}$, and the like of all others.

Rule

Rule 2.

Surd numbers are multiplyed and divided like whole numbers, the Product retaining still the Character of the Root.

That is, $\sqrt{2}$ multiplied by $\sqrt{3}$, produceth $\sqrt{6}$, and so of all others.

NOTE.

Where I shall have occasion (if any be) to speak of a Cubique Root, I shall signe it thus, $\sqrt{c}$, and the Biquadratique Root thus $\sqrt{qq}$.

Rule 3.

To multiply, divide, adde or *Substract the roots of Surd numbers.* And first of

MULTIPLICATION.

Besides that which hath been said in the last Rule above, these roots of Surds may be multiplied and divided, and known by other names, so as sometimes the products, or quotient shall be rationall. First, therefore any square root doubled is the square root of the quadruple, as

$2\sqrt{5}=\sqrt{20}$ and $2\sqrt{20}=\sqrt{80}$.

$3\sqrt{5}=45$. $4\sqrt{5}=\sqrt{80}$. $5\sqrt{5}=\sqrt{125}$.

$2\sqrt{10}=\sqrt{40}$. $3\sqrt{10}=\sqrt{90}$.

$4\sqrt{10}=\sqrt{160}$, and $5\sqrt{10}=\sqrt{250}$, &c, infinitely still multiplying the Numerator, 2, 3. 4, 5, &c. into it selfe, and the product into the surd number, as if $3\sqrt{10}=\sqrt{90}$, it ariseth from 3 times 3 into the surd number $\sqrt{10}$: and the like of all others whatsoever.

For put $\sqrt{a}=\sqrt{10}$, to be multiplied by another

another number, as by $a = 10$, the product is $a\sqrt{a} = 10\sqrt{10}$, which by the first rule is $\sqrt{aaa} = \sqrt{1000}$, that is, the numerator 10 into it selfe making 100, which multiplied again by the surd $\sqrt{10}$, gives $\sqrt{1000}$.

And if it had been at first $\sqrt{a} = \sqrt{10}$, multiplied by any other number, as $e = 3$, the product must by the same method be $e\sqrt{a} = e\sqrt{10}$ that is (by the same reason as the former) $\sqrt{eea} = \sqrt{ee10} = \sqrt{90}$.

And it is plain, that if any Root be multiplyed

$$\text{by } \left.\begin{array}{l} 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\right\} \begin{array}{c} \text{The product shall} \\ \text{be the Root of} \\ \text{the} \end{array} \left\{\begin{array}{l} \text{Quadruple.} \\ \text{Noncuple.} \\ \text{Sedecuple.} \\ \text{Vigintiquintuple.} \\ \text{Trigintisextuple.} \end{array}\right.$$

And so forward infinitely, according to the proportion of the Squares of the Multipliers

Also by Decuplation, as if $5\sqrt{5} = \sqrt{125}$, then $5\sqrt{50} = \sqrt{1250}$. Or if $4\sqrt{4} = \sqrt{64}$ then $4\sqrt{40} = \sqrt{640}$. And (as above) if $4\sqrt{10} = \sqrt{160}$, then $4\sqrt{100} = \sqrt{1600}$.

Also by Subdecuplatiō, if $2\sqrt{10} = \sqrt{40}$, then $2\sqrt{1} = \sqrt{4}$. Or if $5\sqrt{20} = \sqrt{500}$, then $5\sqrt{2} = \sqrt{50}$. And (according to that aforesaid) $3\sqrt{37} = \sqrt{333}$, and $3\sqrt{36} = \sqrt{324}$, that is, the square root of 3 times 3 times 36.

And this may often be of use, not only in numbers but Species, and is therefore to be had in memory by him that would be ready in Multiplication of Surd numbers, or Surd quantities.

Fur-

Furthermore it may be useful to remember that in *Reciprocall Surds* as $4 \surd 5$ and $5 \surd 4$ these two have that proportion one to another as 4 hath to a meane betwixt 4 and 5.

As for example $4 \surd 9$ hath that proportion to $9 \surd 4$ as hath 4 to 6, which is a mean betwixt 4 and 9, for $4 \surd 9 = 12$, and $9 \surd 4 = 18$, but $4'\ \ 6''\ \ 12'\ \ 18''$ or more generally $a \surd e'\ \ e \surd a''\ \ a'\ \ \surd ae''$ for multiply the Meanes, it is $ae \surd a$ and multiply the Extreames it is $a \surd aee$, & divide each of them by a the first is $e \surd a$ the other is $\surd aee$, but by the former part of this rule $e \surd a = \surd aee$ wherefore this is proved,

CONSECTARY.

Hence it is evident that roots of themselves inexplicable may be so multiplied as the product may be rational: for if $\surd 20$, be multiplied by $4 \surd 5$ the product will be $4 \surd 100 = 40$. For $2 \surd 5 = \surd 20$ and $2 \surd 20 = \surd 80$, therefore $4 \surd 5 = \surd 80$, but $\surd 80$ multiplied by $\surd 20$ gives $\surd 1600 = 40$,

I need say nothing of *Division*, for that is no more but by the same steps to go back again, as $\surd 1600$ divided by $\surd 80$ quotient is $\surd 20$. And so of the rest which hath been said in multiplication.

NOTE.

Theſe things being ſo, it will not be hard to find ſome number to compare with any *Surd* number ſo as to make that worke rationall and exprimible which ſeemed not ſo: for there is not any ſurd number can be given which may not by ſome multiplication be made a rationall number: for let it be $\surd\ 5$, $\surd\ 7$, $\surd\ 8$, or any of theſe as $\surd\ 7$ multiplie it firſt by $\surd\ 7$ that produceth 7, but multiply $\surd\ 7$ by any ſquare number whatſoever, as by 4 omitting the ſigne $\surd$, it gives 28, then again multiply $\surd\ 7$ by $\surd\ 28$ it produceth $\surd\ 196 = 14$.

For this is all one as to multiplie one ſquare number by another, which muſt needs produce a ſquare number.

So heer the ſquare number 4 was multiplied by 7 and after by 7, that is by 49, which multipliers cannot produce any other then a ſquare number, to wit 196 *Euclid*.9.1.

And whatſoever hath hitherto been ſaid of Quadratiques, may ſerve for cubiques alſo; due reſpect alwayes had to the degree of the quantity and root, for any $\surd c.$ multiplied by 2 gives $8\ \surd c.$ by 3 it gives $27\ \surd c.$ by 4 it gives $64\ \surd c.$ that is $2\ \surd c.\ 8 = \surd c.\ 64$ and

$3\ \surd c.\ 8 = \surd c.\ 216$, and

$3\ \surd c.\ 27 = \surd c.\ 729$ the proportion ſtill in-

increasing as the Cubes of their nultipliers.

And the like consideration had, this may be applicable to *Biquadratiques*, or any higher order.

And still whatsoever hath been said of multiplication, serves in a retrograde way for division also.

Rule. 4

For ADDITION.

Surd roots are usually added and substaacted by the signs + & — as the square root of 2 added to the square root of 8, sum is $\surd\ 2 + \surd\ 8$ or substracted rest is $\surd\ 8 - \surd\ 2$.

But these may be added into one summe for seeing 8 is quadruple to 2 therefore $2\surd 2 = \surd\ 8$. And the summe is $3\ \surd\ 2$ and the remaine is $\surd\ 2$. Likewise the *reciprocall Surds* $8\ \surd\ 2 = 2\ \surd\ 8$, are capable of addition, substraction, multiplication or division; for they are being added $3\ \surd\ 32$ that is $\surd\ 288$; substracted, $\surd\ 32$; multiplied $\surd\ 4096$; divided $\surd\ 4$: but such as are neither commensurable nor reciprocall cannot be amassed into one summe.

And the summe of the former addition of $\surd\ 8 + \surd\ 2$ being already reduced to $3\ \surd\ 2$ may be yet further reduced to $\surd\ 18$ for $3\ \surd\ 2$ is equall to the square root of three times three

times

times two as hath been more then once ſhewed.

And generally when the Surds given are denominated by numbers in quadruple proportion, as $\sqrt{2}$ to $\sqrt{8}$, and $\sqrt{3}$ to $\sqrt{12}$, &c. the leſſer and the greater twice being added together, as 2 to 16, or 3 to 24, the ſquare Root of the ſum is equall to the ſum of the two ſquare Roots given to be added, that is, $\sqrt{2}+\sqrt{8}=\sqrt{18}$, and $\sqrt{3}+\sqrt{12}=\sqrt{27}$.

The reaſon is, $\sqrt{1}+\sqrt{4}=\sqrt{9}$, which 9 is compoſed of the leſſer once and the greater twice, that is, as often as the $\sqrt{1}$ is contained in the $\sqrt{4}$.

But if the numbers be prime one to another, they muſt be added or ſubtracted by the ſignes $+$ and $-$, for theſe Rules reach not to primes.

And having ſaid this little to acquaint ſuch as are wont to be afr aid of operations where Surds are preſent, with this which will render ſome things eaſie which perhaps ſeemed hard, and others which were hard, leſſe difficult. I will now leave this ragged Subject, and recreate a little with a few eaſie *Propoſitions*; the performing of which may ſerve to recall into uſe and practice that which hath been ſpoken of Solids in the former Chapter.

CHAP. VII.

Prob. 1.

ANy right line being given, to divide it into two parts, so as the Rectangle of the whole and one of the parts; may be to the square of the other part, in such proportion as is betwixt any two right lines given.

Let the right line given be b.
The segment to be squared a.
Then the other Segment is $b - a$.
And let the two lines given be r and s.

Then $bb - ba' \quad aa'' \quad r' \quad s''$.
And $raa = sbb - sba$. per 16. 6. *Euclide*.
That is, $raa + sba = sbb$.

Make $\frac{sb}{r} = d$, and divide all by r.

Then it is, $aa + da = db$. Make $db = ff$.
Then lastly it is $aa + da = ff$. And a is easily found by *Rule* 1. of *Chap.* 2.

And if it had been required to have had the Rectangle + or — some other plain to have had any limited proportion to the square aa, the work had been almost the same, with some small addition.

Prob.

Prob. 2.

To make a Scalenon Triangle, of which the Base, Perpendicular, and proportion of the other Sides shall be given. (I account that the Base which subtends the divided angle.)

Let the base given be b, the perpendicular c; and the proportion of the other sides, as r to s. Of which let r be the lesser.
And for the lesser segment of the base put a:

Therfore by supposition,

$$r'\ s''\ \sqrt{cc+aa}'\ \sqrt{cc+bb-2ba+aa}''$$

So that the squares of them are also proportional,

That is.

$$rr'\ ss'',\ cc+aa'\ cc+bb-2ba+aa''.$$

And by multiplying the means & extreams, It is,

$$ssaa+sscc=rrcc+rrbb-2rrba+rraa.$$

That is,

$$ssaa-rraa+2rrba=rrcc+rrbb-sscc.$$

Make $\dfrac{ss}{r}=x$.

And divide all the æquation by r, Then it is,

$$xaa-raa+2rba=rcc+rbb-xcc$$

Secondly, make $x-r=f$, & $gg=bb+cc$.

Then it is, $faa+2rba=rgg-xcc$.

 Again,

Again make $\frac{rg}{f} = h$, and $\frac{xc}{f} = k$, and divide $faa + 2rba = rgg - xcc$ by f.

Then it will be

$$aa + \frac{2rb}{f}a = hg - ck.$$

Lastly, make $\frac{2rb}{f} = q$, and $hg - kc = mm$

The Æquation finally reduced will be then $aa + qa = mm$, and a may be found by the first rule for square Æquations. *Chap*, 2.

Prob. 3.

Any number being given, to finde two other numbers, so as all the three may constitute a Rectangle Triangle.

Unto the Square of the number given adde unity, the halfe of the summe shall be the hypothenuse, or from the said square take unity, the halfe of the remain shall be the middle side.

For let the number given be a, the square is aa, to which adding unity, the sum is $aa + 1$, the halfe whereof is $\frac{1}{2}aa + \frac{1}{2}$, for the Hypotenuse.

Secondly, from aa take unity, the rest is $aa - 1$, the halfe whereof is, $\frac{1}{2}aa - \frac{1}{2}$ for the middle side. But

But the lesser side (by supposition) is a.
The square of the lesser side is aa.
The square of the middlemost is $\frac{1}{4}aaaa - \frac{1}{2}aa + \frac{1}{4}$

Both those squares are $\frac{1}{4}aaaa + \frac{1}{2}aa + \frac{1}{4}$:
But the square of the hypotenuse, *viz*, of $\frac{1}{2}aa + \frac{1}{2}$ is equall to these, that is, $\frac{1}{4}a^4 + \frac{1}{2}aa + \frac{1}{4}$.
Therefore by the 48. of the first of *Euclide* the Proposition is proved.

COROLLARY.

Hence it is plain, that the two greater sides of any rectangle Triangle differ by unity, for if two Squares differ by 2, their halves differ by 1.

NOTE.

If it be required to have all the three sides in whole numbers, then the lesser side must be an odd number.

Probl. 4.

The difference of the sides of a Rectangle, with the Area and Diagonall in one sum, being given in numbers, to finde out the sides.

Let the difference of the sides be 7
And the Area and Diagonall together 73.
And put the lesser side equall to a.
Then the greater is $a + 7$.
These two multiplied produce $aa + 7a$, which is equall to the Area.

 And

And therefore $73 - aa - 7a$ is the Diagonal

The square of which is $\left.\begin{array}{r}+5329-146aa\\ +aaaa+14aaa+\\ +49aa-1022a\end{array}\right\}$

Which reduced and rightly ordered, Is

$+aaaa+14aaa-97aa-1022a+5329$

Which by the 47. of the first of *Euclide*, is equall to the two squares of the other sides a, and $a+7$, whose squares are aa, and $aa+14a+49$.

That is, $+aaaa+14aaa-97aa-1022a$

$+5329 = 2aa+14a+49$.

That is $+aaaa+14aaa-99aa-1036a$

$+5280 = 0$.

That is, $-aaaa - 14aaa+99aa$

$+1036a = 5280$.

In which æquation, because $aaaa$ hath four dimensions, and the Homogeneall 5280, but four places, the root a cannot consist of more then one place, or figure, which must be found out by trying every one of the nine Digits, if need be, and will be found at last to be 5, therefore the other side is $5+7=12$, the Area 60, and the Diagonall 13.

But if a had been more or lesse then 5 yet (except something else lead a readier way) it is good to try 5 at first, if it be too little then 7, if that also

also too little, then 9, so there will be no need to try the even numbers, 6, 8, &c. for if 5 be too little and 7 too great, it must be 6, the like reason will serve for 8, 4, 2, so that he which guesseth most unfortunately, needs not try above four or five digits, which is no great matter, the like happening sometimes in seeking the quotient in plain division, for no man is sure to guesse right at first.

But that we may exemplifie this in bigger numbers, where a may consist of two or more places.

Let the difference of sides be 71
The area and diagonall together 1177.

Working as in the former example, there will arise an æquation, which being reduced and ordered as before, will be

$$-aaaa-142aaa-2685aa+167276a=$$
$$=1380288.$$

And putting $b+c=a$:

Then the Canon of the resolution will be

$$-bbbb-4bbbc-6bbcc-4bccc-cccc$$
$$-142bbb-426bbc-426bcc-142ccc$$
$$-2685bb-5370bc-2685cc$$
$$+167276b+167276c.$$

To be orderly subtracted from the Homogeneall number given 1380288, as followeth.

The

The number given $+ 1\dot{3}80\dot{2}88$

The first single root $b = 1.$

$-bbbb$	1.0000
$-142bbb$	142.000
$-2685bb$	2685.00
In all	$-$4205.00
$+167276b$	167276.0

Subtract (the difference of $+$ & $-$) $+1252260$

Remains of the number given $+0128028$

The first Root decuplate $b = 10.$

$-4bbb$	4000
$-6bb$	0600
$-4b$	0040
$-426bb$	42600
$+426b$	4260
$-5370b$	53700

Then -105200 is all the $-$

And $+167276$ is all the $+$

Divisor $+62076$ is their difference.

The second single Root $c = 3$

Re-

Remains of the number given $+128028$

$-4bbbc$	12000
$-6bbcc$	.5400
$-4bccc$	.1081
$-cccc$	...81
$-426bbc$	127800
$-426bcc$	.38340
$-142ccc$	..3834
$-5370bc$	161100
$-2685cc$	24165
In all $-$	373800

$+167276c = 501828$
All the $-$ being 373800

The differēce is $+128028$

Which being taken from the remaines of the number given $+128028$, there remains finally nothing, ſo that the given æquation is juſtly reſolved by the Root $b+c=13$.

The leſſer ſide a is therefore 13, to which if the difference given, namely, 71, be added, the middle ſide 84 is thereby compoſed.

Again, if to that middle ſide 84 be added unity, the hypotenuſa of a right angled triangle is compoſed, whoſe three ſides are 13, 84, 85.

The

The Superficies of this Triangle is halfe the parallelogram or rectangle required.

For 84 multiplied by 13, gives 1092 for the area of the rectangle, to which adding 85 the Diagonall, composeth the number 1177, as was required in the Proposition.

COMPENDIUM.

Seeing the two greater sides of any rectangle triangle, exceed one another by unity (as by the former Corollary) the difference betwixt the two lesser sides being given, the difference betwixt every two sides is also given.

So that putting a for the lesser side of the rectangle, the greater side is $a + 71$, and the diagonall $a + 72$, whose square is $+ aa + 144a + 5184$, to which the two squares of the sides, being $aa + aa + 142a + 5041$, are equall:

That is,

$$2aa + 142a + 5041 = aa + 144a + 5184$$

And subtracting from each part

$$aa + 144a + 5041$$

There will remain $+ aa - 2a = 143$.

And a will be found 13, by the second Rule of *Chap.* 2.

RESUMPT.

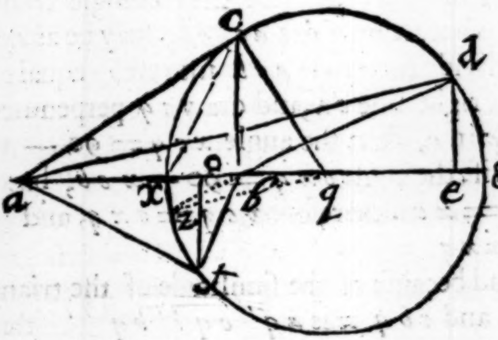
In the second *Probleme* of this Chapter it hath been shewed how upon a Base and Perpendicular

and

and proportion of the remaining ſides given, to deſcribe a Triangle.

It is there to be underſtood of an acute angled triangle, in which the perpendicular falls within the triangle.

Now therefore let it be otherwiſe.



As in this figure, of the triangle *adb*, let there be given, the baſe $ab = b$, the perpendicular $de = c$, and the proportion of the other ſides, namely $bd = r$, and $da = s$, whereas in the ſecond Probleme, the leſſer ſegment of the baſe was called *a*, and the greater $b - a$. Now here the line *be* may be called *a*, and *ae* ſhall be $b + a$, and the reſt of the work will be like that before in *Probl.* 2.

NOTE.

And it may be noted, that if from *a* and *b* (the

(the ends of the line ab) be drawn other binarie lines, how many ſoever, ſo as they hold the ſame proportion as s to r and concur in other points, as c, f, g, &c. Thoſe points are all in the circumference of a circle whoſe center is in the line ab, produced towards g.

For upon ab deſcribe the rectangle triangle abc whoſe two ſides ac, cb, may be as s to r, and divide the angle acb into two equal parts by the right line cx, and draw cq perpendicular to ac in c, then the angle $xcq = 90 - xca$ likewiſe the angle $cxq = 90 - xcb$, but $xcb = xca$ therefore $xcq = cxq$; and $cq = xq$.

And becauſe of the ſimilitude of the triangles acq and cbq it is $aq\ '\ cq\ ''\ bq\ '''$ that is $aq\ '\ xq\ ''\ bq\ '''$.

Now by ſuppoſition $ag\ '\ bg\ ''\ ac\ '\ bc\ ''$

And it hath been proved $aq\ '\ xq\ ''\ bq\ '''$.

Therefore by compoſition alſo, it will be $aq + xq\ '\ xq + bq\ ''\ xq\ '\ bq\ ''$.

But $ac\ '\ bc\ ''\ aq\ '\ xq\ ''$.

And $xq\ '\ bq\ ''\ aq\ '\ xq\ ''$.

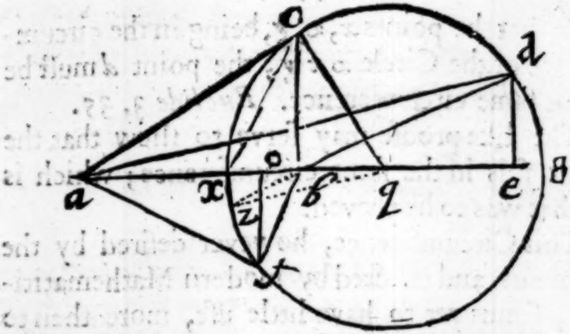
Therefore $ag\ '\ bg\ ''\ aq + xq\ '\ xq + bq\ ''$.

And therefore $ag = aq + xq$. And $xq = gq$.

But it hath been proved that $xq = cq$.

Therefore $xq = cq = gq$.

Againe, making the center q, and the ſpace



ſpace $x\,q$, $c\,q$, or $g\,q$, deſcribe the circle $x\,c\,g$, and prolong $d\,b$ to the circumference in z, and draw $q\,z$, and becauſe it hath been proved above, that $a\,g\,' \;\; b\,g\,'' \;\; a\,c\,' \;\; b\,c\,''$.

That is $a\,g\,' \;\; b\,g\,'' \;\; x\,q\,' \;\; b\,q\,''$ therefore alſo

By diviſion $a\,g - b\,g\,'\,b\,g\,''\,x\,q - b\,q\,'\,b\,q\,''$.

That is $a\,b\,' \;\; b\,g\,'' \;\; x\,b\,' \;\; b\,q\,''$.

Therfore the rectangles $a\,b\,q = x\,b\,g$ *Eu:. 6.16.*

And becauſe by ſuppoſition

$a\,c\,' \;\; b\,c\,'' \;\; a\,d\,' \;\; b\,d\,''$.

And $a\,c\,' \;\; b\,c\,'' \;\; q\,c\,' \;\; q\,b\,''$.

Therefore $a\,d\,' \;\; b\,d\,'' \;\; q\,c\,' \;\; q\,b\,''$, but $q\,c = q\,z$, therefore $a\,d\,' \;\; b\,d\,'' \;\; q\,z\,' \;\; q\,b\,''$ and the angles $a\,b\,d$, $z\,b\,q$ being equal, the triangles $a\,b\,d$, $z\,b\,q$ are equiangular, *Euclid. 6. 6.*

And therfore $a\,b\,' \;\; d\,b\,'' \;\; b\,z\,' \;\; b\,q\,''$ *Eucl. 6.4.* and the rectangles $d\,b\,z = a\,b\,g$: *Eucl. 6. 16.* But it was now proved that $a\,b\,q = x\,b\,g$. therefore $x\,b\,g = d\,b\,z$.

So

So that the points x, z, g, being in the circumference of the Circle $z\ x\ g$, the point d must be in the same circumference. *Euclide* 3. 35.

The like proofe may serve to shew that the point f is in the same circumference; which is all that was to be proved.

This Circumference, however desired by the Ancients, and effected by modern Mathematitians, seems yet to have little use, more then to help the construction of the triangle, which (but now I shewed) may be done without it.

CHAP. VIII.

Of Mixtion.

DEFINITION. 1.

STandard *finenesse*, Is that finenesse which the current Gold and Silver Moneys are made of. In *England* the Gold is 22 Carects of fine Gold, and two Carects of Allay. The silver Monies are made of silver so as the pound weight contains eleven Ounces, two peny weight of fine silver, and 18 Peny weight of Allay.

DEFINITION 2.

If any Ingot be finer then Standard, it is called *better*, if courser *worse*, and this *betternesse* and *worsenesse* is reckoned by *Carects* and *Grains* in

in *Gold*, and by *Peny weights* in *Silver*, and is ſummed by multiplication of the betterneſſe or worſeneſſe in the pound weight, or pound weighrs of the Ingot.

DEFINITION 3.

The Temper is that which when two or more quantities of Liquors, or Herbs, or Minerals, or any thing uſed in Medicine, of differing degrees of Heat, Cold, Drouth, or Moiſture, are mixed together, ſo as the whole Maſſe ſo made by mixing have none of theſe four Qualities.

NOTE.

The *Standard* and the *Temper* differ in this, the firſt reſpecteth but two qualities, to wit, *better* and *worſe*: the lattet reſpects four qualities, namely, *hot*, *cold*, *dry*, and *moiſt*: yet the Mixtor dealing but with two of theſe at once, that is, ſuch two as are oppoſite, as are the two firſt or the two laſt mentioned before, or any two which are alike, as both better, both worſe, both hot, or both cold, may uſe the ſame way in both.

Prop. 1.

If there be two Ingots of equall weight, the one better then Standard by a certain fineneſſe, the other as much worſe, thoſe two Ingots molten together ſhall produce a Fuſion or Maſſe which ſhall be of Standard fineneſſe

Let

Let the first *Ingot* b be better by c.

The Second *Ingot* d worſe by f.

And let it be $b = d$ and $c = f$.

Multiply b by c it gives bc equal to all the betterneſſe of the *Ingot* b. By *Def.* 2.

Likewiſe cf is equal to all the worſtneſſe,

but $bc = df$.

Therefore the whole *Fuſion* $b + c$ is aſmuch better then ſtandard as worſe.

Wherefore it is neither better nor worſe, but juſt Standard fineneſſe.

Prop. 2

If two Ingots to be molten differ in weight, quality, and degree of quality reciprocally, that is, if as the weight of the firſt, to the weight of the ſecond, ſo the degree of worſtneſſe of the ſecond to the degree of betterneſſe of the firſt, the whole fuſion ſhall be Standard fineneſſe.

Let there be quantity b better by c.

And quantity d worſe by g.

And let it be $b\,'\,d\,''\,g\,'\,c\,'$.

Therefore $bc = dg$ *Eucl.* 6. 16.

Namely all the betterneſſe equal to all the worſtneſſe, and therefore the mixture of the maſſe neither better nor worſe. The ſame arguments will ſerve if the *Propiſition* had been in *Liquors*, to prove the mixture to be temperate.

Prop.

Prop. 3.

If there be two quantities of Silver or Liquor, of divers qualities, or divers degrees of the ſame qualitie, if all the betterneſſe or all the worſtneſſe, all the heat, or all the cold be found out by multiplying each quantitie by its qualitie, and taking the difference of them if they be oppoſite; or the ſumme of them if they be alike; that difference or ſumme divided by the ſumme of the quantitie, gives (as ſome call it) the forme reſulting or the degree of betterneſſe, worſtneſſe, heat or cold, of the whole fuſion or mixture.

Let there be quantity b hot in g.

And quantity d cold in h.

Then bg is equal to all the heat of b.

And dh equal to all the cold of d.

If $bg > dh$ then $\frac{bg - dh}{b + d}$ is the degree of the form reſulting, Hot, Or if $bg < dh$, theu $\frac{dh - bg}{d + b}$ is the ſame in coldneſſe.

Now although this is plaine from *Def.* 2. becauſe all the heat $bg - dh$, or the coldneſſe $dh - bg$ of the whole mixture, ariſeth by multiplication of the ſeverall qualities by their reſpective quantities, and therefore that whole heat, or whole cold divided by all the weight of the ſeveral quantities, gives the quotient equal to the degree of heat or cold of any part of the weights, which in reſpect of the whole weight may be called one, which degree of heat here

$\frac{bg - dh}{b + d}$ being multiplied by the whole weight namely by $b + d$ gives $bg - dh$, that is all the heat of all the weight, and therefore $\frac{bg - dh}{b + d}$ is that which we call the forme resulting, and $\frac{dh - bg}{b + d}$ if $bg < dh$. Yet this may be further confirmed by that Rule given by Mr. *John Dee*, in his *Mathematicall Preface* before *Euclid*.

The Rule which there he sheweth is this.

> *What proportion is of the lesser quantity to the greater, the same is of the difference between the degree of the forme resulting and the degree of the greater quantitie to the difference between the degree of the said forme and the degree of the lesser quantitie.*

Here therefore let be $b < d$ for that is free. Also let it be $bg > dh$.

It is to be proved by the said Rule,

that $b' \; d'' \; h' + \frac{bg - dh'}{b + d} \, g - \frac{bg + dh''}{b + d}$

Multiplie the two later by the cōmon denominator $b + d$, the first gives $bg + bh$, the second $dg + dh$.

And

And therfore $b' \; d'' \; bg + bh' \; dg + dh''$.
Multiply the means it gives $dbg + dbh$, likewise the extreams multiplied is $dbg + dbh$: And therefore the Analogisme which was to be proved is true, by the 16. of the 6. of *Euclide*.

In like sort if it were $bg < dh$, and $b < d$, it might be proved, that

$$b' \quad d'' \quad h - \frac{dh + bg'}{b+d} \; g + \frac{dh - bg''}{b+d}$$

Lastly, if g and h were like qualities, that is, both Hot, or both Cold, and $b < d$, It is then to be proved that

$$b' \quad d'' \quad -h + \frac{bg + dh'}{b+d} \; g - \frac{bg - dh''}{b+d}$$

And reduced $b' \; d' \; bg - bh' \; dg - dh''$, Which is manifest.

Example in Numbers.

First, Let it be put $b = 5$, $g = 4$, $d = 7$, and $h = 2$.

Then $bg = 20$, & $dh = 14$, & $\frac{bg - dh}{b+d} = \frac{1}{2}$,

Now because the heat bg is greater then the cold dh, the whole mixture shall be hot, and that heat shall be in the middle of the first degree, and according to Mr. *Dees* Rule it will be

 $5' \quad 7''$.

$5' \ 7'' \ 2\frac{1}{2}' \ 3\frac{1}{2}''$, for the difference between the form resulting which is hot in $\frac{1}{2}$ and the greater quantities degree, which is cold in 2, is $2\frac{1}{2}$, likewise the difference between the lesser quantities degree, hot in 4, and the forme hot in $\frac{1}{2}$ is $3\frac{1}{2}$: So that this is right.

Secondly let be $b = 5, g = 2, d = 7, h = 4$, in opposite qualities h and g: $\dfrac{dh - bg}{b + d} = \frac{18}{12}$,

It will be $5' \ 7'' \ 4 - 1\frac{1}{2}' \ 2 + 1\frac{1}{2}$ (for $\frac{18}{12} = 1\frac{1}{2}$. That is, $5' \ 7'' \ 2\frac{1}{2}' \ 3\frac{1}{2}''$ as before, and the forme resulting cold in $1\frac{1}{2}$ degree.

3 Lastly, let be $b = 5, g = 4, d = 7, h = 2$, in like qualitie, for example both hot

then $\dfrac{bg + dh}{b + d} = 2\frac{10}{12}$ for the forme.

And $5' \ 7'' \ \frac{10}{12}' \ \frac{14}{12}''$ for the Anologisme, exactly agreeing in all cases with Mr. *Dee*.

And this is so plain that it needs not be exemplified in metals, it not being my purpose to write much of them nor of the Standarding of Gold and Silver, because it is so neatly and fully done already in a little Treatise put forth in *Anno.* 1651. by Mr. *John Reynolds* of the Mint.

Yet the Reader may take notice that he which brings but common Arithmetique with him, may by some one or more of these three foregoing propositions, perform any plain Pro-

blemo

bleme that can be required concerning mixtures in valuable metals or liquors. For first

Rule. 1.

If the weight of the masse be not limited, if any quantity with any quality (which exceeds not the degree of the greatest finenesse) be given, a like quantity of the just opposite quality, will cause all to be Standard or Temper.

Rule. 2.

If the quantity of the masse be limited, and the two oppositive qualities given, *then divide the quantity of the masse into two parts proportionall with the qualities, and taking them reciprocally the mixture shall be Standard, or Temper*, by the second Prop.

Rule. 3.

If there be two Ingots of Silver to be molten the first better by a certain difference, and the second also better not by the same difference, *if each weight be multiplied by its betternesse, the two products added together make the betternesse of the whole masse; which being divided by the summe of the two weights, gives the forme resulting of the masse* by the third proposition, which

masse may be made Standard by Allay as followeth.

As the fine silver in the pound Standard,
Is to the forme resulting:
So is the weight of the masse,
To the weight of the Allay.

But this Rule is not pertinent to the mixture of liquors, because in them there is nothing agreed on for Allay.

NOTE.

If the two Ingots molten produce a masse worse then Standard, out of any Silver which is better, a quantity may be limited by the second proposition, to make it Standard

But if there be given the weight of an Ingot worse by a certaine difference; and the weight of the whole fusion be limited, and the finenesse, wheher Standard, or better, or worse; This Rule doth it. *Multiply the weight by the worstness, and divide the product by the betternesse of the Silver to be added, the qnotient shall be the weight of that to be added to make it Standard.* And if it be required to have the fusion better, or worse then Standard (but not worse then the Ingot given) it is easily done by taking more or lesse weight of the fine silver to be added, or of more or lesse finenesse as the case requires, and which needs no more then hath been shewed. If

If the fusion consist of more then two quantities, all that hath been said of two things miscible, is appliable to other miscibles how many so ever, by repetition of the working with two at a time.

Prop. 4

If there be three like solids equal in Magnitude, and differing in weight, the middlemost being composed of some of the matter of the first, and the rest of the matter of the third mixed, if the rectangle made of the weights of the first and third, Minus the rectangle made of the weights of the said first and second, be divided by the weight of the third want the weight of the first, the Quotient shall be eqnal to all the matter of the first (that is to the weight thereof) which is contained in the mixed solid.

Let the first be b, the second c, the third d.

And the weight of the first q, of the second r, and of the third s. And the common magnitude Unity, and make a equal to the weight of all the matter of the first contained in the second, as aforesaid.

And make $q \,'\, 1 \,''\, a \,'\, \frac{a}{q}$:

And $s \,'\, 1 \,''\, r - a \,'\, \frac{r - a}{s}$

Therefore

Therefore $\frac{a}{q}$ is equall to all the said matter of the first, and $\frac{r-a}{s}$ to all of the third in the Mixture, I mean to the Magnitude of it.

And $\frac{a}{q} + \frac{r-a}{s} = 1$

Multiply all by qs (or first by q, and the product by s) it gives $sa + qr - qa = qs$,

That is, $sa - qa = qs - qr$.

But $qs - qr$ is the Divideud required.
And $s - q$ the Divisor required.

And $\frac{qs - qr}{s - q} = a$ by the last Æquation,

And a equal to the weight of the matter in the first contained in the second, wherefore the Proposition is proved.

Example in Numbers.

Put $q = 97$ $r = 73$ and $s = 63$

Then $\frac{qs - qr}{s - q} = 28\frac{9}{17}$, which is all of the matter of b contained in c.

And

And the residue $73 - 28\frac{9}{17} = 44\frac{8}{17}$ is all of *d* contained in *c*.

$$\text{Now } 97' \quad 1'' \quad 28\tfrac{9}{17}' \quad \tfrac{485}{1649}'' = \frac{a}{q}$$

$$\text{And } 63' \quad 1'' \quad 44\tfrac{8}{17}' \quad \tfrac{756}{1071}'' = \frac{s-a}{1},$$

$$\text{But } \tfrac{485}{1649} + \tfrac{756}{1071} = \frac{a}{q} + \frac{s-a}{r} = 1.$$

As it ought to be, the like proofe serves for any Numbers.

Prop. 5.

If there be three like Solids of which the second is composed of divers matters, to wit of parts of the first, and parts of the third, and the three solids equall in weight, but not in magnitude, if the rectangle made of the Magnitudes of the first and third, lesse the Rectangle made of the Magnitudes of the first and second, be divided by the Magnitude of the third want the Magnitude of the first, the Quotient will be equall to all the matter composing of the first, I mean to the magnitude thereof, which is contained in the second Solid.

The proofe of this is the same with the former, *Mutatis mutandis.*

Prop.

Prop. 6

If there be three such Solids as before in the fifth, and the magnitudes of the parts composing found, if the magnitude of the parts of the first composing the second, be divided by the magnitude of the first, the quotient is the weight of those parts.

For the common weight being Unity, As the first magnitude is to its weight, which is unity; So is the magnitude of the parts of the first Solid, to the weight of the said parts. (Not to retein the same form, but diffused in mixture, and compared in minute parts commensurable with the whole.

Let the magnitudes of the intire Solids be f, g, h, and their common weight Unity, and let the magnitude of the parts of the first composing the second, be put equall to e.

Then $f : 1 :: e : \frac{f}{e}$ which $\frac{f}{e}$ is the weight of the said parts composing. This is plain.

Prop. 7.

If there be three Solids, and the first and third composing the second as before, differing all in weight and magnitude: if the rectangle Parallelepipedon made of the weights of the first and third, and the magnitude of the second (all multiplied together) want the rectangle Parallelepipedon made of the first and second (I mean the

weights

weights of them) and the magnitude of the third (all multiplied together) be divided by the rectangle made of the magnitude of the first, and the weight of the third want the rectangle made of the weight of the first, and the magnitude of the third, the quotient shall be the weight of the parts of the first composing the second: which weight multiplied by the magnitude of the first and the product after divided by the weight of the first, this later quotient shall be equall to the magnitude of the said parts.

Let the Solids be in $\left\{\begin{array}{ll} \text{Weight} & f,\ g,\ h, \\ \text{Magnitude} & b,\ c\ \ d, \end{array}\right.$

And f the weight of the first, and put the weight of the parts composing of the first equall to a.

Therefore $f'\ b''\ a'\ \dfrac{b.\ a''}{f}$ that is, as the weight of the whole first to its magnitude, so the weight of part or parts of the said first, to their magnitude.

Now because the weight of the parts of the first composing the second Magnitude g, are a, the weight of the parts composing of the third are therefore $g - a$.

Therefore secondly $h'\ g - a''\ d'\ \dfrac{dg - da''}{h}$

That is, As the weight of the whole third, is to the

the weight of the parts thereof, ſo is the Magni-tude of the ſaid whole, to the magnitude of the parts thereof. So then the magnitnde of the parts of the firſt, more the magnitude of the parts of the third, are equall to the whole magnitude of the ſecond.

That is $\frac{ba}{f} + \frac{dg - da}{h} = c$

Multiply both parts by fh the rectangle of the denominators, it gives

$+ hba + fdg - fad = fhc$

That is $hba - fda = fhc - fdg$

But $fhc - fdg$ is the dividend propoſed.

And $hb - fd$ the diviſor deſired.

And $\frac{fhc - fdg}{hb - fd} = a$ by the laſt equation.

But by ſuppoſition a is equal in weight to the parts of the firſt compoſing the ſecond, wherefore the propoſition is proved, as to the firſt part.

And the ſecond part is manifeſt out of the firſt Analogiſme $f'\ b''\ a'$ $\frac{ba''}{f}$ to wit that $\frac{ba}{f}$ is equal to the magnitude of the parts of the firſt.

Example in Numbers.

Let the weight of the firſt ſolid be 85

of

of the ſecond 60
of the third 54

The magnitude of the firſt 49
of the ſecond 50
of the third 48

And the weight of the parts of the firſt a as before.

Then $85'\ 49''\ a'\ \dfrac{49\,a''}{85}$

And $54'\ 60 - a''\ 48'\ \dfrac{2880 - 48\,a''}{54}$

And therefore $\dfrac{49\,a}{85} + \dfrac{2880 - 48\,a}{54} = 50.$

That is the magnitude of the parts compoſing the ſecond taken together, muſt be equal to the magnitude of the whole ſecond.

Multiply each part by 85 times 54, that is by 4590.

It produceth

$2646\,a + 244880 - 4080\,a = 229500$

That is (reduced) $-1434\,a = -15300.$

And $a = 10\frac{960}{1434}$

Now the rectangle parallelepipedon of 85, 54, and 50, is 229500, from which taking 244800 (which is the rectangle parallelepipedon of 85 60 and 48) there remaines $-$ 15300 for the *Dividend* propoſed.

Se-

Secondly, If from the Rectangle of 49 and 54, which is 2646, be taken the Rectangle of 85 and 48, which is 4080, remains —1434 for the divisor proposed. And (by the last Æquation) $\frac{15300}{1434} = a$, and therefore the Example in Numbers is cleared.

In the Æquation before $hba - fda = fhc - fdg$, the quantity a is easily found by this analogism, namely,

$bb - fd\,'\,hc - dg\,''\,f\,'\,a\,''$, if one make

$$bb - fd = mm \text{ and } hc - dg = nn.$$

And $m\,'\,n\,''\,p\,'''$, for then $m\,'\,p\,''\,f\,'\,a\,''$.

Upon the same way of reasoning which hath been used in this Chapter, is grounded the *Rule of False Position*, and also that of *Alligation*: For if the two degrees of the qualities of any two Miscibles, be called the two false positions, and the two respective quantities of the said Miscibles, be called the two *Errors*, then the degree of the form resulting is the true point sought. For if any one would work by the *Rule of False*, and go the neerest way, he must divide the distance betwixt the false positions into two parts proportionall with the Errors, and the work is thereby done sooner then by the common way of Crosse *Multiplication*.

As if it were required to part 48 in two, and one

one of the parts againe into 3, and the other into 4, so as the thirds of the one may be (in number) quadruple to the fourths of the other. Suppose first 40 and 8, and dividing 40 by 3, quotient is $13\frac{1}{3}$, and 8 by 4 is 2, whose quadruple should be $13\frac{1}{3}$, but is but 8, so the first Error is $-5\frac{1}{3}$.
And putting the second time 30 and 18, the second Error will be found to be $+8$.
Make therefore $8 + 5\frac{1}{3}{}'$ $40 - 30{}''$ $8{}'$ $6{}''$ if this 6 be added to the second position 30 whose error we here worked with (namely with 8) the summe is 36 for the part required, and the other part is 12.

And as for the *Rule of Alligation*, which is to adde all the betternesse and worstnesse of each particular component severally taken into one summe (which there is called the summe of the differences;) And then to worke by this *Anolagisme, viz.*

As sum of all the betternes and worstnes mixed,
Is to the whole Mass, or mixture to be made;
So is any particular betternes or worstnes.
To all that which is to be taken and mixed of that respective quality.

All this being manifest, shall not need any proof.

CHAP.

☞ As before, Page 106, ſo here again, I let the Reader know that the word Magnitude in this Chapter, is to be taken for the number of ſmall parts or atomes of a Body, and not for a line or Superficies.

CHAP. IX.

Of Menſuration.

IN this Chapter I ſhall demonſtrate little, as not intending to write much new, but (for the moſt part) ſuch as hath been already exhibited by *Archimede* and others, yet put here becauſe the Book ſhould not want ſomething for the Reader which hath not read ſuch Authors, and for ſuch as ſtand in need of the thing rather then the Proofe.

If there be a Cube whoſe ſide is $b\,c$, and a Sphere whoſe Axis $u\,e$ is equall to $b\,c$ (which here we put 7,) and an upright Cone $a\,d\,f$, whoſe baſe is $d\,f$, equall alſo to $b\,c$, and its altitude $a\,o = b\,c$.

1 Then

1 Then the ſuperficies of the Cube (being equall to the Square $bcfd$ multiplied by 6 is 294.

2 And the ſuperficies of the Sphere (being quadruple to the Circle $uaeo$) is 154.

3 And the Superficies of the Cone (being made by multiplying the ſide $ad = \sqrt{61\frac{1}{4}}$ by the ſemicircumference $auo = 11$) is 86, and not conſiderably more, to which adding the ſuperficies of the Baſe $38\frac{1}{2}$, the whole ſuperficies of the Cone is 124½

4 And if there be a Priſme, whoſe Baſe and altitude are ſeverally equall to the baſe of the Cube, or of any other rectangular Parallelepipedon, the Priſme is the halfe of the Parallelipipedon in ſolidity. *Eucl.* 12. 7.

5 And if a Piramis inſiſt on the ſame Baſe with the Priſm, having equall altitude, the Piramis is two third parts of the Priſm, or $\frac{2}{6}$ of the Parallelepipedon.

6 The ſolidity of a Cone or Piramis is found by multiplying its altitude by $\frac{1}{3}$ of the area of the Baſe.

The ſolidity of theſe other is found thus.

For the Cube, Multiply the ſide [7] by the ſquare of the ſide [49] it gives the ſolidity of the Cube, which is 343.

For the Sphere, Multiply the Cube of the Diameter [343] by 11, and divide the product by 21, it makes the solidity of the Sphere, which is —— $179\frac{2}{3}$

The solidity of a Cylinder, whose Diameter and Altitude are the same with the Diameter of the Sphere, is made by multiplying the superficies of the base [$38\frac{1}{2}$] by the altitude [7] whereby the solidity is produced == $269\frac{1}{2}$

For the Cone, *Euclide* having proved it to be the third part of the Cylinder, *Euc.* 12. 10. the solidity thereof is ———————— $089\frac{5}{6}$

A Fragment.

The superficies of the Fragment of a Sphere is found by multiplying the superficies of the whole Sphere by the altitude of the fragment, and dividing the Product by the Diameter of the Sphere, and adding to the quotient the superficies of the base of the Fragment.

The solidity of a Fragment (lesse then half a Sphere) is found thus.

From the Semidiameter of the Sphere, subtract the altitude of the Fragment, and by the remain mul-

multiply the area of the Base, and ſubtract the product from that which is made by multiplying the ſemi-axis of the Sphere into the convex ſuperficies of the fragment: Laſtly, divide the reſidue by 3, the quotient ſhall be the ſolidity ſought for. If the fragment be more then halfe a Sphere, ſubtracting this from the whole, the greater fragment is thereby had.

This laſt Rule preſuppoſeth the Axis of the Sphere to be known; but if it be not ſo, it may eaſily be found by the following analogie.

Let the altitude of the fragment be b
the ſemidiameter thereof c,

Make $b' \; c'' \; \dfrac{cc'''}{b}$ and make $f = \dfrac{cc}{b}$

Then it is manifeſt by the 13 of the 6 of *Euclide*, that $b + f$ is equal to the diameter of the Circle, or to the Axis of the Sphere.

It is manifeſt by the former work, that the Solidities of the Cone, Sphere, and Cilinder, being $89\frac{5}{6}$ $179\frac{4}{6}$ $269\frac{3}{6}$ are in proportion one to another as 1, 2, and 3, for the Cone is $\frac{1}{3}$ and the Sphere $\frac{2}{3}$ of the Cilinder, but the ſuperficies of the Sphere and Cilinder are equal excepting both the baſes of the Cilinder

So by that which hath been ſaid afore the Pyramis, Priſme and Cube of equal baſe and altitude, are in ſolidity alſo as 1, 2, and 3.

There may be other parts of a Sphere beſide thoſe which here are called *Fragments*, (not to ſpeak of thoſe which are irregular & Multiform) which are either Cones or Piramids, whoſe baſes lie in the ſuperficies of the Sphere, and their Vertices at the center, the ſolidity of one of theſe is found by multiplying the third part of the Baſe by the altitude, (which here is the ſemiaxis) the product is the ſolidity: Theſe fragments are thoſe which are uſually called Solid angles.

Example.

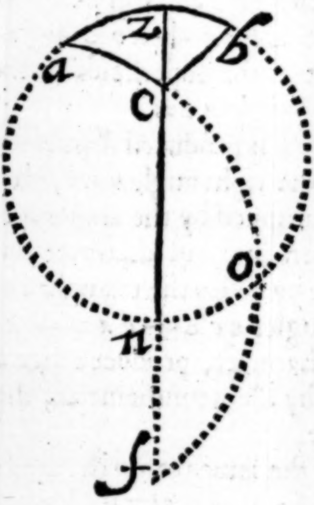
Let there be a Piramis of three ſides, whoſe Baſe is $19\frac{1}{4}$, equall to $\frac{1}{8}$ of the ſuperficies of the Sphere, and the vertex thereof in the center, it is plain enough that this Piramis is the eight part of the Sphere in ſolidity.

Multiply $6\frac{5}{12}$ (the third of the baſe) by the perpendicular $3\frac{6}{12}$, the product is $22\frac{11}{24}$, which is the ſolidity of the Piramis, and multiplied by 8 gives $179\frac{2}{3}$ equall to the whole Sphere.

The like for Cones in this caſe.

If the ſuperficies of the Baſe be not wholly given, if any three things be given (if they be not the three angles) the three angles may yet thereby be found. And then, the Rule (which I had from my learned friend Mr. *John Leeke*) is, *If the exceſſe of the three angles above* 180 deg. *be*

be multiplied by halfe the Diameter of the Sphere, the ſuperficies of any Sphericall Triangle is thereby produced.



This may be thus demonſtrated, let the ſuperficies of the oblique Triangle *a b c* be required, from the greateſt angle *c*, let fall to the baſe *a b*, a perpendicular *c z*, produced towards *f*, and continue the baſe *a b* till it cut *z c*, produced in *n*, and *a c* produced in *o*, then the triangle *n f o* is equall to *a c z*, being equiangular, and having the ſides *c z* and *n f* equall, for either of them is equall to the diameter of the Sphere, or rather a Semicircle want the line *c n*.

 Now

Now if the angle at *a* be multiplyed by the diameter of the ſphere, it makes the ſuperficies, *a c o b z*, likewiſe if the angle *f c o* that is *a c z*, be multiplyed by the ſaid diameter, they produce the ſuperficies *f n c o*. And both theſe ſuperficies are equal to *z b o n* + *a c z* + *f n o*, that is to a fourth part of the ſuperficies of the ſphere plus twice the triangle *a c z*.

But *z b o n* is produced by the diameter multiplied by the right angle *n z b*, wherefore the diameter multipyed by the angles *a c z* + *c a z* is greater then the ſaid diameter multiplied by a right angle by twice the triangle *a c z*, and therefore the angles *a c z* + *c a z* — *a z c* multipyed by the diameter, produce twice the ſuperficies *a c z* or by the ſemidiameter, they produce it juſtly once.

And by the ſame reaſon the angles *b c z* + *c b z* — *b z c* multiplyed by the Radius produce the Superficies *b c z*, which added to *a c z* make the whole *a b c*.

But the angles

a c z + *c a z* — *a z c* + *b c z* + *c b z* — *b z c*.

Are the ſame as the angles

a b c + *b c a* + *b a c* — 180*d*. which is the difference whereby the three angles given exceed two right angles. So that this exceſſe multiplyed by Radius as aforeſaid, produceth the ſuperficies of the whole Triangle at firſt given, namely *a b c*, which was to be demonſtrated.

Many

Many more such things might be taken out of *Archimedes*, as to measure the Superficies made by revolution of a Spirall line, and others, which seldom occurre to any vulgar use: And for that cause, and also because the recitation of them would not benifit the other sort of Readers which know them already, I medle no further, but will leave this Subject after one short Rule for measuring Hogsheads or Barrels, which is this.

From the area of the Circle of the greater diameter, multiplied by the length of the Vessel, subtract the area of the lesser, multiplied also by the length of the Vessell, and take the third part of the difference from the greater area multiplied by the length as before; the rest is the content sought for, in such measure as was the length, of the Vessel. That is Inches, if the Scale were so: of which 231 make a Wine Gallon, and 288 $\frac{3}{4}$ a Beer Gallon, or rather an Ale Gallon, according to some accounts, but not yet resolved fully.

Otherwise thus.

Let the square of the greater diameter in Inches be *b b*.

The square of the lesser diameter in Inches *c c*.

The length in Inches *d*.

And let the content in Inches sought for be *a a a*.

Then it will be $aaa = \frac{22dbb + 11dcc}{42}$

Example.

Let it be $b = 6$
$c = 3$
$d = 10$

Then	$22dbb =$	7920
And	$11dcc =$	. 990
	In all	8910

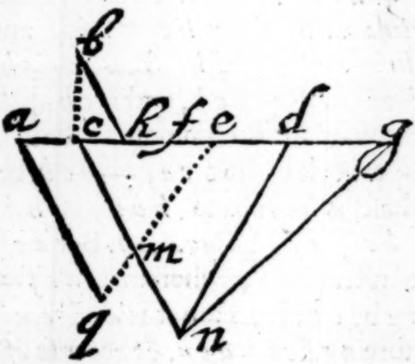
Divide 8910 by 42 the quotient is $212\frac{1}{7}$, which is equall to aaa the content in inches, which was required. The very same number will come forth if one work by the former way, putting circumference to Diameter, as 22 to 7. But although this should be exactly true in one Vessel (which cannot be proved because of the irregularity of the Vessel) it would not be so in others, because of the irregularity (or diversity) of this irregularity.

In Mr. *Spidals* Extractions there are many Propositions of worth, and all undemonstrated, I will therefore in this place bestow a Demonstration on one of the hardest of them, which is this.

Let it be required to divide any Triangle, as

cng

cng from a point without the Triangle as *q*, into two parts, of which one part shall have any proportion to the whole, given between two right lines, as here the lines *cd* and *cg*.



By the point *q* draw *qa* parallel to the neerer side *cn*, cutting *gc*, produced, in *a*.

Make aq' cn'' ac' cf'',

And ac' bc'' cf'''.

And part *cf* into two equall parts in the point *h*, and draw the Diagonall *bh*, And make $he = bh$. Lastly, draw the line *qe*.

Then I say the Triangle *cng* is divided by the line *qe* into two parts *cme* and *mnge* so, that as *cme* is to the whole *cng*, so is *cd* to *cg*.

DE-

DEMONSTRATION.

Foraſmuch as the right line cf is divided into two equall parts, in the point h, and to it is added another line fe, therefore the Rectangle $cef + hfh$ is equall to the Square heh (by *Euclide* 2. 6.) but $he = hb$, and therefore $hbh = cef + hfh$. But $hc = hf$, therefore $hbh = cef + hch$, and $hch + bcb = hbh$: Take away hch common to both Æquations, Then it is plain, that $cef = bcb$, becauſe either of theſe is equall to $hbh - hch$: ſo then $ce' \; bc'' \; ef'''$, *Euc.*6.17. But $ac' \; bc'' \; cf'''$ by conſtruction, wherefore $acf = cef$, and ſeeing by conſtruction it is $aq' \; cn'' \; cd' \; cf''$, therfore $aqfc = den$, & becauſe $ef' \; cf'' \; ac' \; ec''$ therefore by compoſition $ce' \; cf'' \; ae' \; ce''$, but $ae' \; ce'' \; aq' \; cm''$, becauſe the two laſt are parallels, *Eucl.* 6.2. And therefore $aq' \; cm'' \; ce' \; cf''$, and $aqfc = mce$, but alſo $aqfc = ncd$, as is proved before, and therefore the Rectangles $ncd = mce$, and alſo the halves of them are equall, namely the triangles ndc and mce, but the Triangles ndc and ngc have that proportion as have their Baſes, cd and cg, wherefore $mce' \; ngc'' \; cd' \; cg''$, and the line me is drawn from the point q which was to be proved.

I know this is demonſtrated by others already, but I may aſwell inſert a Demonſtration diffe-

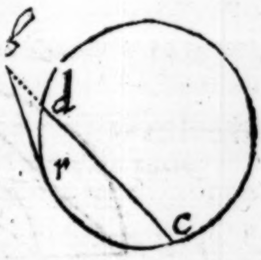
ring

ring, as Mr. *Spidall* might write the same Proposition without proof.

Some *Corollaries* might be deduced from this Proposition, by considering the various analogies therein, which, I leave to the invention of the Reader.

Now if it were required to draw a line from a point given without a Circle given, through the Circle, so as to cut off an arch equall to an arch given, that may very easily be done in this manner.

The Circle *c r d* being given, and a point without it at *b*, let it be required to draw from *b* to the inner Circumference at *c*, a line *b d c*, so as that the arch thereby cut off *c r d*, may be equall to an arch given, for example, to 120 degrees.



From *b* draw the tangent *b r*, and make $br = c$
$cd = b$
$db = a$

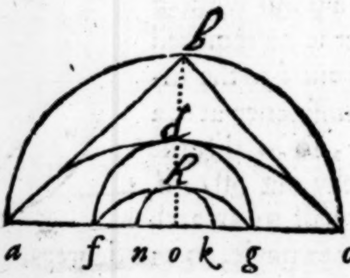
Then it will be $b + a\,'\,c\,''\,a\,'''$, *Euclid* 3.36.
Therefore $aa + ba = cc$, *Euclide* 6. 17.
Wherefore *a* may he found by the first Rule for plain Æquations, *Chap*. 2.

CAAP.

CHAP. X.

THe ſuperficies of an *Ellipſis* may be eaſily found as neer the truth as that of a Circle, becauſe it hath been proved by diverſe to be a mean proportionall between the two Circles deſcribed ſeverally upon the diameters of the Ellipſis, and it is almoſt axiomatically evident by meer inſpection of the figure following.

And therefore it is as eaſie to give an Ellipſis in any proportion to another Ellipſis, as to deſcribe any Ellipſis at all.



As for *Example*, Let the greateſt Diameter of the Semi-ellipſis *a d c*, be *a c* = 28. then the ſemicircle deſcribed thereon ſhall be *a b c* = $\frac{88}{2}$ and let the leſſer diameter of the ſaid Ellipſis be 2 *do* or *fg* = 14.

Laſtly, let it be required to deſcribe an Ellipſis which ſhould be to the Ellipſis *a d c*, as 1 to 4.

Upon the line *a c* from *o* both wayes, ſet off

f o

fo and *go*, each of them equall to *do*, and divide *do* into two equall parts in *h*, then describe the Ellipsis which shall passe by the three points *f*, *h*, *g*, I say that the Ellipsis *f h g* is to the Ellipsis *a d c* as 1 to 4.

For seeing the Circle *a b c* is to the Circle *fdg* in diameter double, therefore $abc = 4fdg$, and of what parts soever *abc* is 16, of those *fdg* shall be 4.

And seeing the Ellipsis *a d c* is a mean betwixt them, the said Ellipsis is 8 of the same parts.

Again, by the same reason the Circle *fdg* is quadruple to the Circle *nhk*.

Therefore of what parts soever *fdg* is 4, of those *nhk* shall be 1.

And seeing the Ellipsis *f h g* is a mean betwixt them, the said Ellipsis is 2 of the same parts.

But the Ellipsis given *a d c* is 8.

And 2′ 8″ 1′ 4″, which was to be done.

In like sort having duly proportioned the Diameters of Circles, may be made Ellipses, in any proportion one to another, or in any proportion to a Circle given.

And the works may be proved by induction, as this also might have been, for seeing the circle $abc = 616$, the Circle $fdg = 154$, the Ellipsis *a d c*, a mean betwixt them, must be $= 308$.

Again,

Again because the Circle $f d g = 154$.
And the Circle $n h k = 038\frac{1}{2}$
The Ellipsis $f h g$ being a mean betwixt them must be $= 77$.
But 77′ 308″ 1′ 4″, &c.

NOTE. 1

Herein I make use of that proportion which is betwixt 22 and 7 for the Circle to the Diameter for easinesse in accompt, small and whole numbers being also better attended and understood sooner by the Reader; and for no other cause: the more exact proportion being as 355 to 113, or (which is more used) as 360 to $114\frac{5915492}{10000000}$.

NOTE. 2

Hence it is manifest that the Content of the Lunula *a d b c* comprehended by the Circle *a b c* and the Ellipsis *a d c*, (being according to this account halfe the Circle *a b c*, that is 308.) As also the mixed figures *a d f* and *c d g* (being here the residue of the Semicircle *f d g*, to the Semi-Ellipsis *a d c*) may be found out as exactlie as the Superficies of a Circle. with which, until a further discovery, we must be content. And I have here noted it, to shew that investigation is not yet to be contemned, as if the thing sought were

were (not onely impoſſible but) uſeleſſe, when ſo many neat Propoſitions might thereby be ſtarted, as would (although not ſo abſolutely neceſſary for preſent uſe, yet) delight the modeſt eye with the novelty.

NOTE. 3

Moreover if the ſaid Lunula *a d b c* were compoſed of two Circles, there might be a rectiline figure given equal to the ſuperficies thereof; That is, if the ſuperficies of a Circle *a d c*, were double to the ſuperficies of the Circle *a b c*, the lines *ab*, *bc*, being drawn, the triangle *a b c*, would be equal to the Lunula *a d b c*; as might be proved if it were not eaſie, and well enough known already.

So that ſome figures of crooked lines, either differing in kind, or in quantity may be equalled with Rectiline figures, or numbers; And yet where Circles of equal quantity include any Lunula or other figure, this cannot yet be done; So thinne is that Curtain which is drawn between us and our deſires.

NOTE 4.

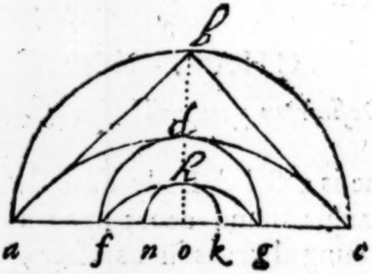
Whereas in the former figure, the making of the Ellipſes, *a d c*, *f b g*; is not ſhewed; this may be here uſefull to ſome: and it is as follows.

The

The greater Axis of the Ellipsis being equall to the diameter of a Circle *a b c*, namely, to the right line *a c*, the other Axis to be taken at pleasure according to the occasion; having here assigned the line *d o* for the halfe of the lesser Axis, draw from the Circle to the diameter *a c* perpendiculars as many as you pleaſe. Then lastly, dividing each perpendicular into two parts proportionall with *b d* and *d o*, in certain points, if by those points (of which the more, the better) a line be drawn with an even hand, that line shall passe also by the point *d* and be the Ellipsis required.

Otherwise, and more for Mechanick use.

Having chosen the two Axes *a c*, and 2 *d o* and made them cut one another into two equal parts, and at right Angles in the point *o*; take the halfe of *a c*, and apply it both ways from the point *d*, to the Diameter *a c* in *x* and *y*, then in the points *x* and *y* which points *x* and *y* are called the burning points, fix two pinns, of Iron, or wood (as the greatness of the Plain shall require) And upon the Plane place a string that compassing both pinns shall reach just to the point *d*, or *c*, (for all is one) and there fasten the ends of the string together by a knot or otherwise, at which knot holding a Pencill, and carrying the Pencill round upon the Plane, about the pinns with



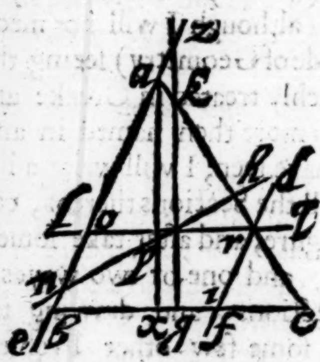
with the ſtring allways ſtraight, the Ellipſis (whoſe halfe is *adc*) ſhall be thereby deſcribed.

Moreover (although I will not meddle much with this kinde of Geometry) ſeeing theſe things are already richly treated in Greeke and Latine, and not much more then named in any Engliſh Book that I have ſeen, I will write a little here of a *Cone*, and all the Sections thereof, comprehended in one figure, and after take ſome principall Definitions, and one or two wayes of deſcribing the Sections, and drawing tangents to them, and ſome few other Problems out of *Claudius Midorgius*, not word for word, but as it ſhall ſeem convenient here.

CHAP. XI.

Definition generall of a Cone.

A Cone is a Solid Body made by turning a Triangle round about upon a Plain, one side remaining alwayes in its place, the second side describing a Circle upon the plain, the third side describing the superficies of the Cone in the



aire; the first side is the Axis, the Circle, the Base, and the Vertex of the Triangle is the Vertex of the Cone. As let the plane Triangle *a b c* represent half a Cone, which is made by the Motion of the Triangle *a b x*, about the Axis *a x*, the side *x b* describing a Circle upon the plane *b c*, of which *b c* is the Diameter and doth here represent the base of the Cone, which added to the Superficies

perficies described by *a b*, moving about in the aire, composeth the whole superficies of the Cone: of which *a*, is called the Vertex, or top. And if the Angle *a x b*, be (as here) a right Angle, the Cone is called an *upright*, otherwise *Scalenon*, either of them may be cut by Sundry Planes, as first by a Plane passing through the Vertex *a*, and perpendicular to the plane of the Base *b c*, and this Section in upright Cones is an Isosceles Triangle, in the other Cones, a Scalenon Triangle, except it be *a b* = *b c*, for then in upright Cones the Section is an Equilateral Triangle, in Scalenon Cones an Isosceles.

1. Now let this Triangle *a b c* represent halfe the Cone as aforesaid, and then if a plain, as *e b o a z* touch the Cone all along from *b* to *a*, and make right angles with *b c* the diameter of the base, and again, another plain *f d* parallel to *e b o a z* cut the semi-cone *b a c*, the section *i r* in the superficies of the Cone is halfe a *Parabola*, the other halfe underneath, if the Cone be supposed entire, and is not to be projected *in plano*.

2 Again, if the Semi-cone *b a c* be cut by another plain *g k z*, parallel to the Axis *a x*, the section in the superficies of the semicone, to wit *g k* shall be halfe an *Hyperbola*, and the like for the other halfe underneath, if the Cone were supposed intire, and further, whatsoever plain cutting the Semicone as aforesaid being

produced shall concur with the plaine *b a* produced towards *z*.

Thirdly, If the said Semi-cone be cut by a plain *n p h*, neither of the former wayes, nor parallel, nor subcontrary to the base, the line in the superficies, namely *n h* is a *Semiellipsis*.

Subcontrary position is that where two like triangles are joyned at an equall (and then verticall) angle, yet have not their bases parallel.

Lastly, if it be cut by a plain *l o r q* parallel to the plain of the base, the section *o r* is a Semicircle.

Definition 1.

Opposite Sections are two Hyperbola's in opposite superficies cut by the same plain.

Definition 2.

The Vertex of a Section is a point in the greatest curvature thereof, but more generally the point where any diameter cuts the Section, and where the Axis cuts is called the highest Vertex.

Definition 3.

Any two lines applyed within the Section, and equidistant, are called *Ordinately applied*, in respect of some diameter which divides them into two equall parts.

De-

Definition 4.

Any line drawn so as it cuts the section, and divides the Ordinates into two equall parts, is called the *Diameter* of the Section, and if it divide them as aforesaid, and at right angles, it is the *Axis*, and so much of the Axis or diameter as lies betwixt the Vertex and any ordinate is called (in respect of that ordinate) the *intercepted Axis*, or *intercepted Diameter*, and those two diametets which mutually divide lines applied in the Section and parallel to the Diameters, into two equall parts are called *Conjugate diameters*, of which, as likewise of the oppsite Sections, I intend to say no more in this Tract.

Definition 5.

The transverse Diameter of an Hyperbola, is a right line in the intercepted diameter continued without the Section, and is equall to the double of that line intercepted betwixt the Vertex and the center, and connects the Vertices of opposite Sections: In an Ellipsis; or Circle, it is any whole Diameter: in the Hyperbola and Ellipsis, if it be the continuation of the Axis, or the Axis (in the later) it is called the *transverse Axis*. But the Parabola whose Diameters are all equidistant, hath no transverse Diameter, nor any center.

Definition 6.

The *Center* is a point where all the Diameters meet.

Definition 7.

The *Figures* of Hyperbola's, and Ellipses, and Circles are paralleligrams included between the transverse Diameter, and the contiguous Parameter, of which those are called transverse sides, and these Coefficients by some.

Definition 8.

The said *Parameter* is a right line drawn to touch the Section at the end of the intercepted Diameter, to which all the Ordinates are parallel, and according to which they are compared, and valued, which is therefore called *juxta quam possunt*: and if it be contiguous to the Axis, it is called *the right Parameter.*

Definition 9.

The ambilicius, focus, or burning point in the Parabola, is a point in the Axis distant from the Vertex by a fourth part of the right Parameter.

But in the other two Sections, the burning points are assigned in the Axis of either Section, distant from either end of the transverse axis by the space of a right line that is the square root of the fourth part of the figure produced by the said transverse axis, and the right Parameter, which applied to the transverse axis is in the Hyperbola *excedent* in the Ellipsis *deficient.*

The

The ſame points in any Ellipſis whoſe diameters or diameter are given, may eaſily be found by the mechaniqne way of deſcribing an Ellipſist alittle before ſhewed. Wherein alſo it is plain that theſe points are as it were Centers proper to the generation of the Section.

CHAP. XII.

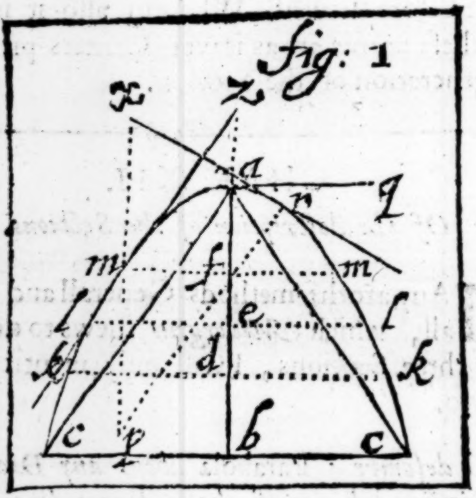
Of the deſcription of the Sections.

MAny are the methods Generall and Speciall, which *Midorgius* ſhews to deſcribe theſe three Sections, I will only mention one or two.

I. *To deſcribe a* Parabola *about any Diameter given with one of the ordinate lines.*

Let the Diameter given be ab, and let bc be one of the ordinate lines applied unto it, by which the angle abc being given, joyn a and c by the right line ac, And let ab be divided into as many parts as you pleaſe, and through every ſuch diviſion draw right lines parallel to bc, and produce them, and make $dk = \sqrt{bc}$ in dg, likewiſe $el = \sqrt{bc}$ in eh, and $fm = \sqrt{bc}$ in fi, and ſo of all the reſt, and the points c, k, l, m, a, &c. ſhall be all in the ſame Section, ſo that a line drawn with an even hand by all the ſaid points, ſhall be by the firſt

Prop. of the ſecond of *Midorgius*, the Parabola required,



And *b c* on the one ſide, is equall to *b c* on the other ſide, becauſe by ſuppoſition, that, and all the parallels to it *k d*, *l e*, *m f*, &c. are thoſe lines which are called Ordinates, or Ordinately applyed, and ſo *a b* in reſpect of *b c*, alſo *a e* in reſpect of *l e*, &c. are the intercepted Diameters, or if the angle *a b c* were a right angle, the intercepted *Axes*. *Def*.4.

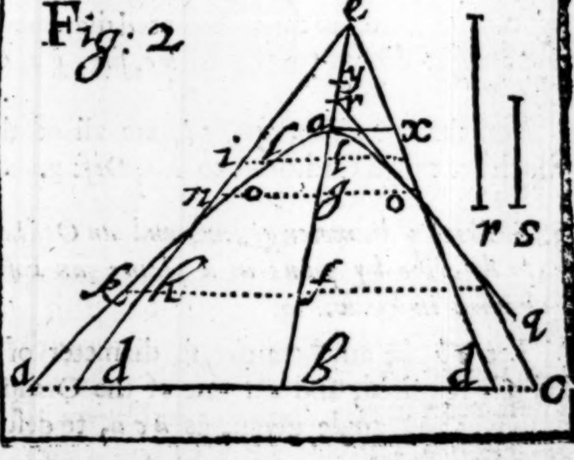
And if you make *a d*′ *d k*″ *a q*‴, and draw *a q* parallel to *d k*, then *a q* ſhall be the contiguous Parameter in reſpect of the intercepted diameter *a d*, and ſo may the Parameter by *a b*, or any

any other diameter given, be found, and therefore the Parameter *a q* only being given, the Parabola by points may easily be described.

2. *About any Diameter, and one Ordinate line, to describe an Hyperbola known in kinde, in a plain by points.*

Let *a b* be a diameter of the Hyperbola, and *b c* an Ordinate to it, comprehending the angle given, *a b c*, and let the Section be of such a kinde, as that the transverse diameter to the contiguous parameter may be as *r* to *s*.

Fig: 2



Make

Make $ab' \ bc'' \ bd'''$, And $s' \ r' \ bd' \ be''$ and joyn the points *d* and *e*, and in the line *ab* take points how many soever, and by them points *f*, *g*, *t*, *&c.* draw lines parallel to *bc*, as *fh*, *gn*, *ti*, *&c.* the more, the better, and making the triangle *ded* compleat, produce these parallels both wayes to the sides *de*, in the points *h*, *n*, *i*, *&c.* Lastly, making *fk*, *gd*, *tl*, *&c.* the square roots of the rectangles *afh*, *agn*, *ati*, *&c.* the points *k*, *o*, and *l*, shall be in the Hyperbola required: *per* 5. *of* 2. *Midorg.* And therefore a line drawn with an even hand to passe by the said points, shall be the Hyperbola required.

And the transverse Diameter thereof is the line *ae*. Wherefore, by the Proposition, if you make it $r' \ s'' \ ae' \ ax''$, then *ax* shall be the contiguous parameter to the intercepted diameter *ba*, supposing it drawn parallel to *bc*, *fk*, *go*; *&c.* *Def.* 8.

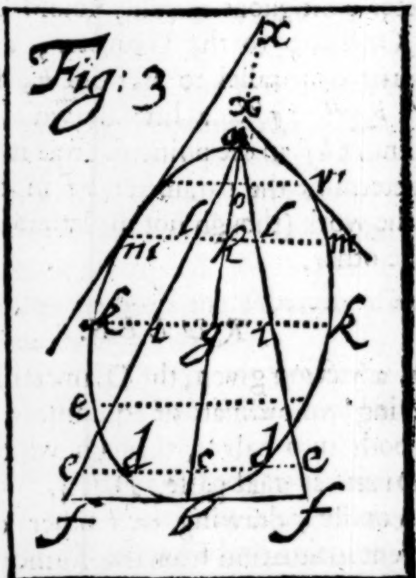
And the lines *fk*, *go*, *tl*, are all of those which are called Ordinates to *ba*, *Def.* 3.

3. *About a diameter given, and an Ordinate, to describe by points in a plain; an Ellipsis known in kinde.*

Let *ab* be any transverse diameter of the Ellipsis required, and *cd* one of the Ordinates drawn at any angle given, as *acd*, to describe as aforesaid; &c.

Make

Make the rectangle *b c e* equall to the square of *c d*, and by *a* and *e* draw the line *a e*, and produce it to *f*, that is, so far as till it meets with *b f*, being made parallel to *c d*, and in *a b* take other points *g h*, through which draw lines parallel to *c d*. Lastly, to every Rectangle *b g i*, *b h l*, &c.



make squares equall, as the square of *g k* equall to the first, and of *h m* to the latter Rectangle, and so as many as you please; the points *m*, *k*, *d*, shall be (by the 3. of the second of *Midorgius*) in the same Ellipsis of which *a b* is the transverse diameter, and *b f* the contiguous parameter,

wherefore

wherefore a line drawn with an even hand by those points *m*, *k*, *e*, shall be the Ellipsis required.

COROLLARY.

Hence it is evident, that having the transverse diameter of an Hyperbola, or an Ellipsis, the parameter contiguous is easily found by applying any Ordinate to the Diameter, as *k g*, and drawing a parallel to it from *b*, for making *b g′ k g″ i g‴* a line drawn from *a* to *i* shall meet *b f* in the point *f*, so as it shall thereby determine the parameter *b f* in the Ellipsis, and the work (though not the letters) is the same in the other.

RULES.

In a Section given, the Diameter is found by applying two ordinate or equidistant lines divided both into halves, through which divisions the Diameter must passe. *Def.* 4.

Secondly, drawing two other equidistants different in situation from the former, and dividing them as aforesaid, you have another diameter.

Thirdly, produce both, and where they concur is the center of the Section. *Def.* 6.

Fourthly, Produce them still (in the Hyperbola) till the space betwixt the Vertex and the Center be doubled, that doubled space is the trans-

transſverſe diameter: the Vertex is here meant at large, for that point of the Section through which the diameter paſſeth. *Def.* 5.

Fifthly, Having the center, an arch of a circle, any where within the Section, biſected, gives a point by which from the center muſt paſſe the *Axis. Midor.* 1.54.

I have ſhewed already how the burning points may be found in an Ellipſis. *Def.* 9.

Sixtly, In the Hyperbola let the tranſverſe axis be *b*, the right parameter *c* produced till it make *x* equall to a mean betwixt them, biſect this mean in *a*, an arch drawn from *a* to the axis (the center being the center of the Section) ſhall there give the point deſired. *Midorgius* 1.58.

The burning point of the Parabola is obvious out of the 9. *Def.*

To finde the Axis of a Parabola.

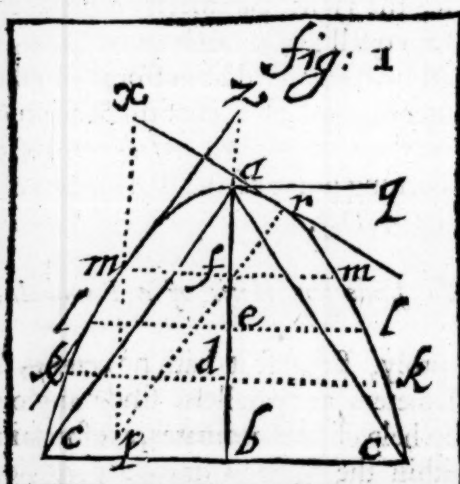
Seventhly, Becauſe it hath no center, but all the Diameters are parallels, finde any one diameter by help of two ordinates, as aforeſaid, and to it within the Section draw a perpendicular, which being produced both wayes juſt to the Section, divide into two equall parts, and through the point of that diviſion, draw a line parallel to the diameter found before, that parallel line is the axis required: The thing is ſo eaſie it needs no Example.

CHHP.

CHAP. XIII.

To draw a tangent to any point assigned in any Section, or from any point without the Section.

NOt to trouble this little book with two many Figures, let the first Figure *viz.* of the *Parabola* be here resumed, which may serve by supposing the Diameter *a b* to be the Axis.



First let it be required to draw a line to touch the *Parabola* in the point *m*, and from *m* draw *m f* perpendicular to the Axis: produce the Axis *b a*, to *z*, making *a z* = *a f* and from *z* to *m*, draw the pricked line *z m*; the said

ſaid line *z m*, is by 55 of the firſt of *Midorgius* the Tangent required.

If the point *m*, had been coincident with the point *a*, a perpendicular to *b a*, in *a* had been the Tangent, *per*. 17 of the firſt *Ejuſdem*.

Now let there be a point given without the Section, (not in the Axis) at *x*, from which let it be required to draw a line to touch the Section.

From *x* draw *x p* parallel to *a b* cutting the Section in ſome point, as here at *m*.

And draw the Tangent *z m*, as aforeſaid and make $m\,p = m\,x$.

And from *p*, draw a line parallel to *z m*, cutting the Section in *r*, and draw *x r*, then *x r* ſhall be the Tangent required by the ſaid 55 of the firſt.

Secondly, let it be required to draw a Tangent to any point in the Hyperbola *d a c*, which ſhall be repeated here alſo, wherein let the Diameter of the Section *a b* be ſuppoſed to be drawn, and the line *g o* any ordinate, and the point *o* to be touched by a right line to be drawn as follows.

Having found the Center *y*, as is ſhewed in the former Rules, make $y\,g\ '\ \ y\,a\ ''\ \ y\,r\ '''$, laſtly from *r* to *o* draw the line *r o q* for the Tangent required *per ditto* 53. 1.

And ſo by converſion of the work, if the point *r* were given without the Section in ſome Diameter or Axis, there might from thence be drawn a right line to touch the Section in ſome place, as here it doth at *o*.

NOTE.

NOTE.

If the point to be touched were in the Section, and in the vertex *a*, then by finding another diameter, another Vertex comes in place, and *a* in reſpect of this other Diameter will be a point in the Section, and a tangent to it as eaſily drawn as to *o*: As may be ſeen in *Fig.* 2.

Thirdly let the Ellipſis *a k b*, be here repreſented, and let it be required from any point given in, or without the Section, to draw a Tangent.

Firſt in the Section at *m*. And from *m* to the other limb of the Section, draw here alſo any line as *m m*, and divide it into two equal parts in the point *h*, and finding the center *g*, draw by *g*, and *h*, the Diameter *a b* and produce it, making *g h*′ *g a*″ *g x*‴: laſtly, from *x* draw *x m* for the tangent required.

Secondly, If the point *x* had been given without the Section, and required from thence to draw a Tangent to the Section in the point *r*, or where it falls, by converſion of the work. Make *g x*′ *g a*″ *g o*‴ ſo have you the point *o*, from which a parallel to *b f* gives the point *r*, where a line drawn from *x* ſhall touch the Section.

The working of theſe things in an Ellipſis is the ſame as in the Hyperbola, only ſeem unlike to them that conſider not fully, becauſe the center

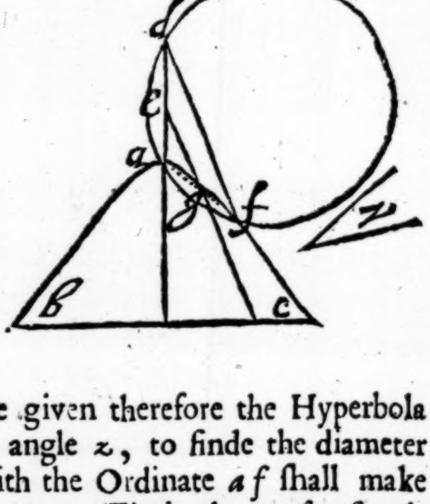
center and transverse diameter of the Ellipsis lies within, and of the Hyperbola without the Section.

And if *h*, or any point within a Section be given, and required through it to draw an Ordinate, that may be easily done, because it must be parallel to a tangent at the Vertex *a*.

Any Section given, to find that diameter thereof, which shall make an angle with the Ordinate to it, equall to an angle given.

If first the Section given be a *Parabola*, finde any diameter, and from the end or vertex thereof,

draw a right line to the Section, making an angle with the said diameter equall to the angle given, to which if a parallel through the middle of the other right line be drawn, that parallel is the diameter required.



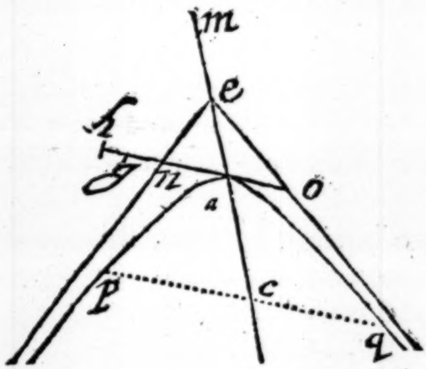
Let there be given therefore the Hyperbola *b a c*, and the angle *z*, to finde the diameter *e g*, which with the Ordinate *a f* shall make the angle *e g a* = *z*. Finde the transverse axis *a d*, and the center *e*, and upon *a d* describe (by the 33. of the 3. of *Euclide*) a portion of a Circle *d f a* capable of an angle equall to *z*, then draw *d f* and *a f*, and through the middle of *a f* draw *e g* the diameter required.

The work is altogether the same in an Ellipsis, only the lesser axis is to be used. *Midor.* 3.67.

Any

Any Hyperbola being given, to finde the Asymptoti.

Finde any diameter of the Section, as *a c*, and the center *e*, and the transverse diameter *a m*, and by the Vertex *a* draw the parameter *a g* contiguous to the diameter *a c*, that is, touching the Section in the Vertex *a*, and unto the rectangle or parallelogram *m a g*, make the Square or



Rhombus of *a h* equall, and divide *a h* into two equall parts in the point *n*, then the right line *e n* drawn and produced shall be one of the *Asymtotes*, then ptoducing *h a* untill *a o* be equall to *a n*, the line *e o* shall be the other *Asymtote*, as appears by *Prop.* 57. *Lib.* 1. of *Midorgius*, Which he demonstrotes out of Propositions of his own Book thus.

Because *a h*, toucheth the Section, it is equidistant to the Ordinates, *per Coroll.* 2, *ad* 17 *primi*, But to the Rectangle or Parallelogram *m a g*, that is to the figure comprehended of the two sides *m a*, and *a g*, is made equal the Square or Rhombus of *a h*, and *a n*, is half of *a b*, therefore the square or Rhombus of *a n*, is equal to a fourth part of the Square or Rhombus of *a h*, that is to the quadrant of the figure *m a g*, *and therefore by the* 38. *of the first and Coroll. to it*, by conversion it may be shewed that the right line *e n*, drawn from the Center and produced how far soever shall never meet with the Section *b a c* and by the same reason, and because *a n* = *a o*, *e o* drawn from the Center shall doe the like, &c.

From hence it appears, that the Asymptotes are lines drawn from the center of the Section; and produced, so as that inclining toward the section still more, shall never be coincident therewith.

More for the Parabola. Numerically.

Let the base be given in Numbers 20, that is, of what lenght soever, let it be parted into 20 equal parts;

And at any inclination to it, let there also be given a diameter; which divide into 100 parts.

And through all the other 9 divisions of the Semi-

Semi-base, draw lines equidistant to the Diameter shortening them in this proportion, *viz.*

Of such parts as the Diameter is 100 let the next be 99, the next 96, the next 91, the fourth 84, the fift 75, the sixt 64, the seventh 51, the eighth 36, the ninth 19. A line drawn with an even hand by the ends of these lines shall be a *Semiparabola.*

The said Numbers are made thus, 10 in 10. 11 in 9. 12 in 8. 13 in 7. 14 in 6. 15 in 5. 16 in 4. 17 in 3. 18 in 2. and 19, in 1. *Prop. 62. lib. 2.*

And they differ just as the square Numbers immediately succeeding to Unity, *viz.* 1, 4, 9, 16, 25, 36, 49, 64, 81, 100. &c, by the quantity of the odd numbers intercepted, as 1, 3, 5, 7, 9, 11, 13, 15, 17, &c.

Which is the same proportion by which the degrees of Velocity of the falling of any thing toward the center of the earth are increased, as *Galileo* hath sufficiently proved in his Dialogues.

And therefore the course of every *Projectile* or thing shot from Gun or Bow may easily be proved to be a Parabolical line.

And the making a Rectilone figure equal to a *Parabola* might be facilitated from hence, if it were not needlesse, the thing being already often done.

Moreover it is to be noted, that the equidistant lines thus drawn, may represent squares, because they differ as the Square numbers doe.

For an Hyperbola, Numerically.

The burning points and transverse axis being given, the Vertex is also given. Let the transverse axis be 80, the distance of each burning point 20 of the same parts, the said points *a* and *b*, the center *a* space 23, and the other center *b*, and space 103, describe arches, which shall cut where the Section is to passe, and so describing from the center *a* other arches, 34, 57, 100, and from the center *b*, with distances 114, 137, 180, other arches, so as the distances from *b* may exceed 100, as much as the distances from *a* exceed 20.

Those arches of Circles shall intersect, and thereby give points by which the Hyperbola is to passe, by the 26. of the 2. of *Midorgius*,

For an Ellipsis, Numerically.

The burning points and Vertices being given (as they were before) the Ellipsis also may be described by numbers as followeth, let the one burning point be at *a*, the other at *b*, and let the diameter be *z*, the distance betwixt *a* and *b* let that be *x* equall to 100, and let it be

$\frac{z - x'}{2}$ — x'' $36'$ $100''$, Therefore also

$z = 132$

$z = 232$, wherefore making the center b at ſeverall ſpaces (more then 16, and leſſe then 116, of ſuch parts as z is 132) as 110, 97, 81, &c. deſcribe arches. Again, making the center a with diſtances 22, 35, 51, and others, ſtill the correſpondent complements of the former diſtances to 132, draw other arches, which ſhall cut the former in points whereby the Ellipſis deſired muſt paſſe, by the ſaid 26 of the ſecond. And it is plain from the generation of an Ellipſis, ſhewed in the inſtrumentall way before in this Book: for the ſtring which deſcribes it is alwayes equall to $z + x$, that is, 232, and ſo is $100 + 110 + 22$ and $100 + 97 + 35$, &c. wherefore this is evident.

And thus they that like this laſt way better, may accompliſh the Section by number.

Moreover, put the diameter of a Parabola

$b = \frac{66\frac{2}{3}}{64}$ of an inch *ferè*.

And let the whole baſe (inclined to the Diameter at angle 84 *ferè*) be $c = \frac{150}{64}$.

Laſtly, Let the perpendicular from the Vertex to the baſe be $d = \frac{64}{64}$

Multiply $\frac{150}{64}$ by $\frac{64}{64}$ the Product is $\frac{9600}{4096}$

Of which two thirds is equall to the superficies of the Parabola, and is $\frac{6400}{4099}$

Of these parts the middle Parallel which was before 75 (when the diameter was supposed 100) is $\frac{50}{64}$ which doubled is $\frac{100}{64}$ that is $\frac{6400}{4096}$ as before.

So that in this case the residue of the Rectangle or Parallelogram, when the superficiall content of the Parabola is taken from it, and the length of the middle parallel, are both denominated by the same number, but this is left to the Reader to try by a Figure delineated by himselfe.

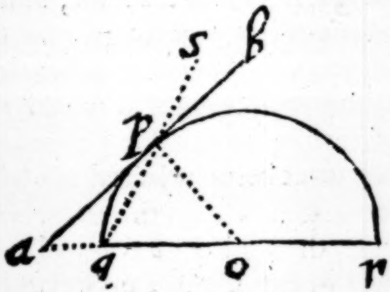
But what use might be made of this (if it were further urged) either in naturall or artificiall numbers, I leave at this time also to the Readers inquiry and study.

Here it may be noted, that a line being drawn to touch a Section in any point, if from that line in that point be raised a perpendicular, that perpendicular is said to cut the Section in the said point at right angles.

For Example, let the right line *a b* touch the Ellipsis *q p r* in the point *p*, and let *p o* be made perpendicular to *a b* in *p*, I say *p o* is usually said

to

to cut the Section *q p r*, at right angles. For, if any line drawn from the diameter *q r* to the Section, may divide it at right angles, let *o p* be supposed to do so, and *a b* a tangent in *p*, as before.



Now if the angle *a p o* be a right angle, all is proved, if not, draw the line *q s* by the point *p*, to cut *p o* at right angles. It is manifest that *q s* shall cut the Section in *p*, because it cuts the tangent there.

Wherefore the same right line *p o* cuts two lines *q p r*, and *q p s* at right angles in the point *p*, namely, where those two lines cut one the other, which is absurd.

And into like absurdities will that opinion lead one which affirms that any crooked line can make any angle with any line whatsoever which toucheth that crooked line.

For although *Clavius* against *Peletarius* and others may say that the angle of *Contact* (as they call it) made between a Circle and his Tangent is lesse then any acute

angle

angle made of right lines, yet ſeeing it is not divisible into parts *aliquotas* or *aliquantas*, which can have any other meaſure then the whole, that is, that each of them is leſſe then any acute angle made of right lines, for ths cauſe, and becauſe it ſeems improper language to give the name of an angle to any ſpace lying betwixt two lines, which although infinitely produced would never meet, I refuſe to call it ſo.

Which ſpace being rejected as nothing, or at moſt non-angular, then the angle *apo* being a right angle, the angle *qpo* is equall to it, and is a right angle in any Circle, or Section of a Cone, or any other crooked line, how much ſoever composed.

For herein it is the ſame with them, as with a Circle, namely, that in the point of any contaction, the angles on both ſides immediately begun are equall.

CHAP. XIV.

I Shall here adde a little to ſhew the reſolution of ſuch Problemes, which ſeeming to require two unknown points at once, are without help of a Conique Section (in lines) inexplicable. And other Problemes may happen higher then theſe infinitely requiring four mean proportionals, or five or ſix. Or to divide an angle into five

five, ſeven or eleven parts, and appearing in Æquations of five, ſeven, or eleven dimenſions, as need requires.

I will begin with the moſt ſimple of theſe, Namely:

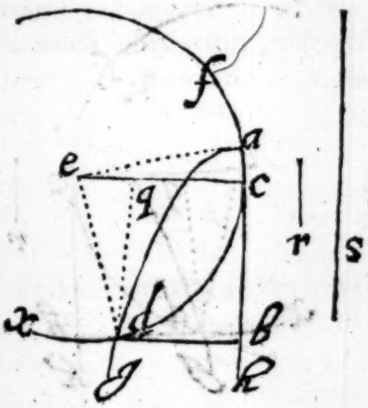
Probl. 1.

Betwixt any two right lines given, as r and s, to finde two mean proportionals.

Put *a* for the leſſer mean required.

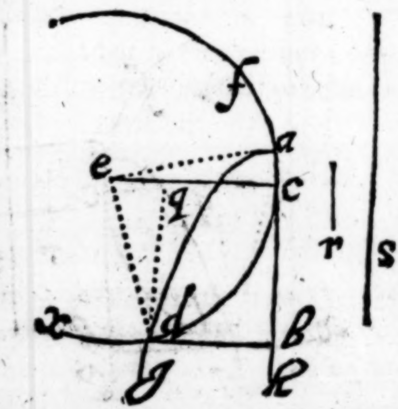
And $r < s$ Then $r\ r'\ aa''\ a'\ s''$, *Euc.* 6.20.

And $aaa = rrs$ *Eucl.* 6.16.



Deſcribe the Semi-parabola *a d g*, ſo as *r* may be equall to the right parameter of it, which may

may be done by *Chap.* 12. hereof and in the axis ah make $ac = \frac{1}{2}r$, and from c raise the perpendicular $ce = \frac{1}{2}s$. And making the center e, and Radius ea describe the circle $fadx$, cutting the section in d, and from d let fall a perpendicular to the axis ah in b, then the lines bd, ba are the two means required. For make $r = 1$, and $bd = a$, then because of the *Parabola* $ba = aa$, for the ordinate db is a mean betwixt the parameter which is unity, and the intercepted diameter ab, *Chap.* 12. And it rests only to be proved that s, or twice ce is equall to aaa, for $1' \; a'' \; aa''' \; aaa''''$ that is $r' \; a'' \; aa''' \; s''''$ Draw dq, parallel to ba. Likewise draw de.



Then $qe = \frac{1}{2}s - a$, for $qc = db = a$. and $ec = \frac{1}{2}s$, likewise dq (that is bc) is $aa - \frac{1}{2}$.

the

the squares of which two, are, $\frac{1}{4}ss - sa + aa$, and $aaaa - aa + \frac{1}{4}$. which together are $aaaa - sa + \frac{1}{4}ss + \frac{1}{4}$. equal to the square of de: *Eucl.* 1. 47. but $de = ae$, and the square of ae is $\frac{1}{4}ss + \frac{1}{4}$. And therefore also it is $aaaa - sa + \frac{1}{4}ss + \frac{1}{4} = \frac{1}{4}ss + \frac{1}{4}$.

That is subtracting from each $\frac{1}{4}ss + \frac{1}{4}$ the residue is $aaaa - sa = 0$. or $aaaa = sa$ that is $aaa = s$, which was to be proved.

This and all other solid æquations not transcending the biquadratique order, are explicable (as *Des Cartes* saith) by a small portion of any of the three Sections. Yet seeing he holds the Parabola the most convenient I make use of that also, and of his Examples in this former and the next succeeding Probleme, aswell because a Parabola is much easier fitted to the worke required, as also for that the demonstration thereby is not so anxious as by the other.

Prob. 2

Now secondly let it be required, to divide any rectiline angle given into three equall parts, as the angle bag.

Suppose it already done by the lines ae, af, and draw the chord bg, and also ec parallel to fa, lastly draw be.

Now

Now because of the similitude of the Triangles bae, bde, and ced, (for the angles are $adg = aeb = bde = ecd$) it it $ab'\ be''\ de'''\ dc''''$.

Put $bg = b$ and $be = a$. And let the Radius ba be Unity.

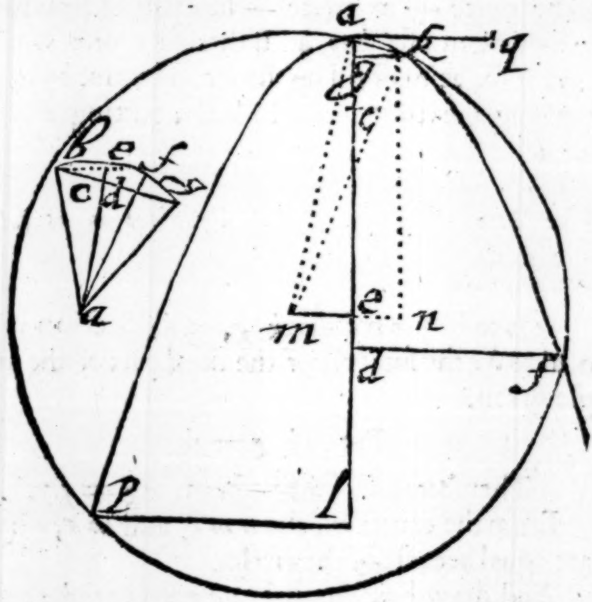
Then $de = aa$ and $dc = aaa$.

And because $bg + dc = 3be$.

Therefore $b + aaa = 3a$, or $3a - aaa = b$ Which Æquation *Pitiscus* hath also in making of the Sines.

Now suppose the Parabola akf drawn so as aq the right parameter may be equall to ba, that is Unity, and the part of the axis ac may be equall to $\frac{1}{2}$, and $ae = 2$, then from e raise em perpendicular to the axis, and equall to $\frac{1}{2}b$, and upon the center m, and the space ma, describe the Circle $akftp$, which shall cut the *Parabola* on that side remote from m, in two points k f, from which perpendiculars in g and d, shall be true roots of this Æquation, of which kg is the subtense of the third part of the arch required, and is equall to be, that is to a, and fd is the subtense of the third part of the complement thereof to a circle, and if on the same side where m is, as from the intersection at p be let fall pl perpendicular also to the Axis, then pl, is a false root of this Æquation, and equall in Magnitude to both the true ones, that is $pl = fd + kg$.

But



But if in stead of $-aaa+3a=+b$, as in this Example it was, the æquation had beene $-aaa+3a=-b$, then the true roots had fallen on the side where the center *m* was, and the false root on that part remote from the said Center.

NOTE.

This Æquation $-aaa+3a-b=0$ is naturally without the second terme *aa*, which is the cause that it hath the false Root not discern-

ced

ed by twice + or twice — ſucceeding, as hath been ſpoken of *Chap.* 4. If therefore one would have it ſo, he muſt fill up the ſecond term, by augmenting the root never ſo little, putting $e - x = a$.

The Demonſtration of this Problem is as followeth.

It is to be proved that kg, in the Section is equal to be the ſubtenſe of the third part of the angle given.

Put $kg = y$.

Then becauſe of the Section. $ag = yy$.

From the center m, draw mk and ma, which are equal becauſe of the circle.

And draw kn, parallel to ae, and produce me to it in n.

Then it is $kn = ge = 2 - yy$.

The ſquare therfore of kn, is $4 - 4yy + yyyy$.

And mn, being equall to y plus halfe the ſubtenſe bg, call bg by the ſingle letter, b, as before, then $mn = y + \frac{1}{2}b$, the ſquare of which being $yy + by + \frac{1}{4}bb$, add to it the former ſquare of kn, that is $4 - 4yy + yyyy$, it makes $+4 - 4yy + yyyy + yy, + by + \frac{1}{4}bb$, equall to the ſquare of the Hypothenuſal mk.

Againe the ſquare of ae, is 4, and the ſquare of me, is $\frac{1}{4}bb$, which two ſquares are equall alſo to the

the square of mk, because $mk = ma$. Therfore

$$4 - 4yy + yyyy + yy + by + \frac{1}{4}bb =$$
$$= 4 + \frac{1}{4}bb. \qquad \text{That is,}$$

$$-3yy + yyyy + by = 0.$$

That is, (by adding on each part $3yy$, and subtracting $yyyy$) $+3yy - yyyy = by$.

Or lastly (dividing all by y) $+3y - yyy = b$.

But this æquation is alike graduated and like affected as the first æquation $+3a - aaa = b$.

Wherefore $y = a$.

But $a = be$ and $y = kg$.

And therefore $kg = be$. Which was to be proved.

In like sort it might be proved that fd is a true root of the æquation $3a - aaa = b$ (in the first figure) and the subtense of the third part of the complement of the angle given bag to a Circle.

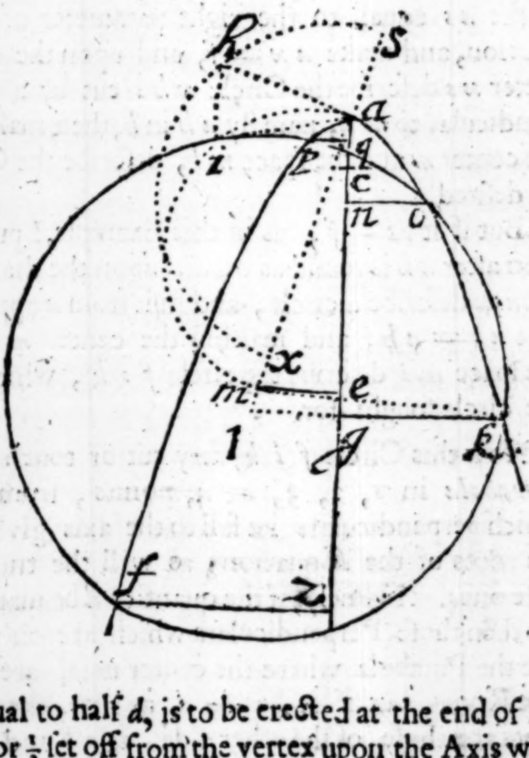
And by such working one may finde it evident that when a Circle cuts a Parabola in points, how many soever (the Vertex excepted) perpendiculars let fall from all those points to the axis, are all the severall roots of one and the same æquation. Nor hath that æquation any more roots then those perpendiculars to the axis.

NOTE. 1.

In the æquation $+aaa - bca = bbd$, the construction differs somwhat from the former, for b being reputed unity, if c as here be signed with — the axis of the Parabola must be produced from the point c in the axis within the Section distant from a by $\frac{1}{2}$ beyond the vertex, till the continuation be equal to $\frac{1}{2}c$, and at the end thereof raise a perpendicular equal to $\frac{1}{2}d$, at the end of that is the center of the circle desired. And according to this method may any æquation, not above biquadratical be resolved, after by taking away the second term (if there be any) by the second Rule of Chapter the fourth it is reduced to such a form as this, $aaa * bca = bbd$, if the quantity unknown hath but three dimensiõs: or if it have four then thus $aaaa * bcaa * bbda = bbbf$. Or else taking b, for Unity, then thus $aaa * ca = d$, and thus $aaaa * caa, * da = f$ the signes $+$ and $-$ are here omitted: for they must be supplied as the nature of the Æquation requireth.

NOTE. 2.

Note that in this breviate the line b, is that which was ba in the example of trisection, and that which was r or unity in the example of two Meanes: Also the line c is that which in the former example of trisection was $2ce$, or 3. And if this quantity be nothing, then the perpendicular equal



equal to half d, is to be erected at the end of half b, or $\frac{1}{2}$ set off from the vertex upon the Axis within, but if c have any length, then at the distance of $\frac{1}{2} c$ from that end, upon the axis. And this which hath been said is enough for all Cubiques.

Prob. 3.

But where the equation is $aaaa - caa + da = f$ so placed as here, if there be $+ f$ and the *P*robleme be to find the value of the root a, then producing ma towards a,

Make as equall to the right parameter of the Section, and make $ax = f$, and upon the diameter xs describe the Circle xhs cut by a perpendicular to ma, namely ah in h, then making the center m and the space mh, describe the Circle desired.

But if it be $-f$, as in this Example I put it, then after ah is found as before: upon the diameter am describe a circle, and in it from a apply a line $ai = ah$, and making the center m and the space mi describe the circle fik, which is the circle sought for.

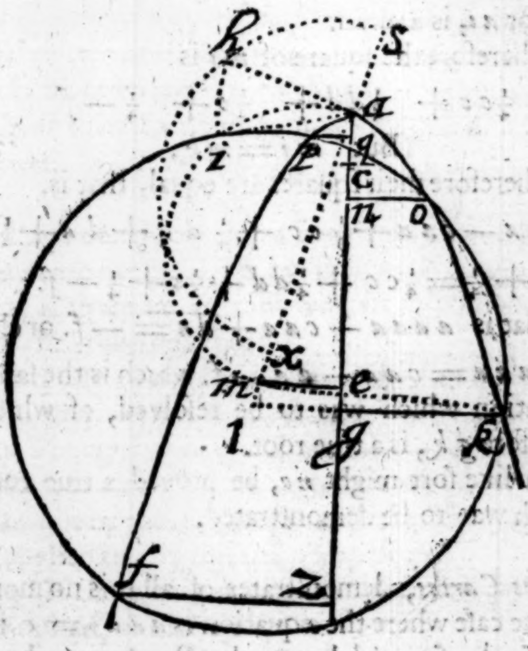
Now this Circle fik may cut or touch the *Parabola* in 1, 2, 3, or 4, points, from all which perpendiculars let fall to the axis give all the roots of the Æquation, as well the true as false ones. Namely, if the quantity d be marked — then those Perpendiculars which are on that side the Parabola where the center m is, are the true Roots, but if it be $+ d$, as here, the true roots are those of the other side, as gk and no, and those of the center side as fz, pq, are the false.

DEMONSTRATION.

Put $ce = \frac{c}{2}$ and draw me perpendicular to ag, and gl equall and parallel to it: lastly,

Put

Put $gk = a$, then $ag = aa$, and taking from it ae, that is $\frac{1}{2}c + \frac{1}{2}$, then $ge = aa - \frac{1}{2}c - \frac{1}{2}$, whose square is $aaaa - caa - aa + \frac{1}{4}cc + \frac{1}{2}c + \frac{1}{4}$.



And because by construction $gl = \frac{1}{2}d$, therefore $kl = a + \frac{1}{2}d$, and the square of it is $aa + da + \frac{1}{4}dd$, which added to the former square of ge, it gives the square of km, that is $a^4 - caa + \frac{1}{4}cc + da + \frac{1}{4}dd + \frac{1}{2}c + \frac{1}{4}$.

Again, the square of ae is $\frac{1}{4}cc + \frac{1}{2}c + \frac{1}{4}$.

To which adding the ſquare of me, that is $\frac{1}{4}dd$ the whole is the ſpuare of ma, to wit $\frac{1}{4}cc + \frac{1}{4}dd + \frac{1}{2}c + \frac{1}{4}$.

But the ſquare of ab, that is ai, is equal to f becauſe $sa = 1$ and $xa = f$ between which ab, or ai, is a mean.

Therefore the ſquare of mi is

$$\frac{1}{4}cc + \frac{1}{4}dd + \frac{1}{2}c + \frac{1}{4} - f$$

But $mi = mk$.

Therefore their ſquares are equal, that is.

$$aaaa - caa + \frac{1}{4}cc + \frac{1}{4}dd + da + \frac{1}{2}c + \frac{1}{4} = \frac{1}{4}cc + \frac{1}{4}dd + \frac{1}{2}c + \frac{1}{4} - f.$$

That is $aaaa - caa + da = -f$. or elſe

$aaaa = caa - da - f$. which is the ſame æquation which was to be reſolved, of which therefore gk, is a true root.

In like ſort might no, be proved a true root which was to be demonſtrated.

Des Cartes, demonſtrates of all this no more but the caſe where the æquation is $aaaa = caa - da + f$, and leaves the Reader to pleaſe himſelf in finding proofes for the reſt : I have choſen this caſe, to demonſtrate, & have demonſtrated the caſes of the two Meanes, and triſection not onely becauſe ſome Readers may be as much pleaſed to have this done to their hand as left to doe themſelves : but alſo that all might ſee

that

that the generall way of demonſtrating all ſorts of caſes, depends on theſe two things; firſt that the right parameter of the *Parabola* being always Unity, if any of the *roots* be put equal to a, the intercepted diameter will be always aa. Secondly, there may be ever found two ſquares equal to two other ſquares, and either the firſt two, or ſecond two equal to the ſquare of Radius. By help of theſe two things may any caſe hereof be proved.

I will conclude with a Breviat of ſuch equations as are not reſoluble by Ruler and compaſſe.

1 If there be as many vowels as conſonants, and the vowels unequal.

As, $ae - da = db$, $ae + da = db$

Or, $-ae + da = db$.

2 Though but of two dimenſions and in feweſt termes, as $ae = bb$, though ſuch are ſolvable yet it may be by infinite ways, and therefore cannot be applied to any limited *Propoſition*.

3 If there be but one Vowell, but cubically multiplied, or higher.

As, $aaa = bbc$. Or, $aaaa = bbbc$.

Where the Æquation being already in the leaſt termes, and not to be brought down by any common diviſor nor the homogeneal bbc, reducible to any ſolid more regular, as, to fff, it is irreſoluble.

CHAP.

CHAP. XV.

HAving said (in the conclusion of the former Chapter) that the Æquation $ae + da = db$ is (as by right lines and Circles only) irresoluble, I will here shew a *Probleme*, by resolving whereof the said æquation will be happened on, which is this following.

Probl. 1.

In any rectangle (b d c a) given, from an angle in it [c] to draw a right line [c f] cutting one opposite side [b d in o] and concurring with the other [b q] produced in [f,] so as the intercepted line [f o] may be equall to [z] any other right line given.

Put $bd = b$. $cd = d$

$do = a$ and $bf = e$

And

And because the Triangles bof, doe, are like, Therefore it is $b - a'\ a''\ e'\ d''$,
And $db - da = ae$, that is, $ae + da = db$.

So we are quickly come to the Æquation required, which æquation having as many unknown quantities (as a, e) as known (to wit, b & d) is hitherto uselesse.

That the Probleme therefore may be solved, we must work another way, and bring it to a Solid Æquation, by making (for more convenience) $cd = b$. $fo = c$. and $bd = d$. and $bo = a$.

Then $d - a'\ a''\ b'\ \frac{ba''}{d-a}$ and $\frac{ba}{d-a} = bf$,

And the square thereof $\frac{bbaa}{dd - 2da + aa} + aa$

is equall to cc, by the 47. of the 1. of *Euclide*.

That is, $aa + \frac{bbaa}{dd - 2da + aa} = cc$

Multiply all by the denominator $dd - 2da + aa$

It makes

$ddaa - 2daaa + aaaa + bbaa =$
$= ddcc - 2dcca + ccaa$, That is,
$aaaa - 2daaa + bbaa - ccaa + ddaa$
$+ 2dcca = ddcc$.

Make

Make $bb + dd - cc = ff$, then ff shall be signed $+$ because hereby supposition it shall be $bb + dd > cc$. And the æquation will e, $aaaa - 2daaa + ffaa + 2dcca = ddcc$.

Expunge the second terme which is $- 2daaa$, by the second Rule of the 4. Chap. And because the Rule is not fully exemplified there in the operosity thereof, I will here work it at large. Because $aaaa$ hath foure dimensions.

Therefore make $4'\ \ 1''\ \ 2d'\ \ \frac{1}{2}d''$

Again, because the first and second term have different signes, therefore put $e + \frac{1}{2}d = a$ *Chap. 4. Rule 2.*

The new Æquation arising thereof will be.

$$\left.\begin{array}{r} + e^4 + 2de^3 + \frac{6}{4}ddee + \frac{4}{8}ddde + \frac{1}{16}d^4 \\ - 2de^3 - 3ddee - \frac{6}{4}ddde + \frac{2}{8}dddd \\ + ffee + dffe + \frac{1}{4}ddff \\ + 2dcce + ddcc \\ - ddcc \end{array}\right\} = 0$$

The homogeneal $- ddcc$, is here put on the same side with the rest, because (for the present) it seems better to stand so, that it may be the last term, in relation to that which is gone before *Chap. 4. Sect. 4.* of the second Rule.

In this last æquation, it is manifest that the second term $2deee$, is (through contradiction of $+$ and $-$) abolished, as was required.

And

And now becauſe the queſititions root e muſt be found by help of a Parabola, as before in the like caſe was uſed, it is neceſſary to reduce the æquation to ſome ſuch form as hath been ſhewed before, in the Note of the former *Chap.*

Firſt therefore to reduce the third term, becauſe $d > f$, and $+\frac{6}{4}dd$, taken from $-3dd$ reſts $-\frac{6}{4}dd > ff$, make $\frac{6}{4}dd - ff = gg$, ſo all of the third term ſhall be $-ggee$.

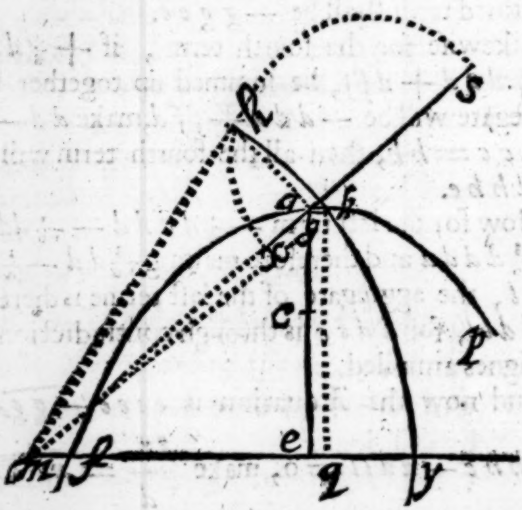
Likewiſe for the fourth term, if $+\frac{4}{8}ddd - \frac{6}{4}ddd + dff$, be ſummed up together the aggregate will be $-ddd + ffd$, make $dd - ff + 2cc = hh$, then all the fourth term will be $+dhhe$.

Now for the laſt term $-\frac{1}{4}dddd - \frac{1}{16}dddd = \frac{3}{16}dddd$ and therefore making $\frac{3}{16}dd - \frac{1}{4}ff = ll$, the aggregate of the laſt terme is thereby $-ddll$, for $ddcc$ is through contradiction of the ſignes annulled.

And now the Æquation is $eeee - ggee + dhhe - ddll = 0$, make $\frac{gg}{d} = m$ and $\frac{hh}{d} = n$, and $\frac{ll}{d} = p$, then the æquation will be $eeee - dmee + ddne - dddp = 0$ and making $d = 1$, then the æquation fully reduced and rightly prepared is $+eeee - mee + ne - p = 0$. (In reducing this or the like conſider *Chap.* 5. *Note.* 2.)

Or

Or $eeee = mee - ne + p$, which is altogether the ſame with that in the former Chapter, and the working of it is there ſhewed. Except only becauſe there the quantity f is ſigned —, and here the like quantity p is ſigned +, I ſhall (although this caſe only is demonſtrated in *Des Cartes*) here demonſtrate it thus.



Deſcribe the *Parabola* $f\,a\,p$, according to the parameter d, (that is $a\,s$) & let $a\,e$ be the axis, & make $a\,c = \frac{1}{2}d$, $c\,e = \frac{1}{2}m$, and at right angles at e make $m\,e = \frac{1}{2}n$, and draw the line $m\,a\,s$, making $a\,s = d$, and $a\,x = p$, and upon $x\,s$ as a diameter deſcribe the ſemicircle $x\,h\,s$, and from a to h raiſe

raise the perpendicular $a\,h$, cutting the Circle in h, and with radius $m\,h$ describe the arch $h\,k\,y$, cutting the Section in k, and from k let fall a perpendicular to $m\,e$ produced in q, and draw the lines $m\,k$ and $m\,h$.

DEMONSTRATION.

To prove $g\,k = e$

Suppose it done, and because $k\,g = q\,e = e$, and $m\,e = \frac{1}{2}n$, therefore $m\,q = \frac{1}{2}n + e$, and the square of it is $\frac{1}{4}n\,n + n\,e + e\,e$.

And because $a\,e = \frac{1}{2}d + \frac{1}{2}m$, and $g\,k = e$, & because of the *Parabola* $g\,a = e\,e$, therefore also $e\,g$ or $q\,k = \frac{1}{2}m + \frac{1}{2}d - e\,e$, & the square thereof is $+\frac{1}{4}m\,m + \frac{1}{2}m\,d + \frac{1}{4}d\,d - m\,e\,e - d\,e\,e + e\,e\,e\,e$. That is (because d is equall to Unity) $+\frac{1}{4}m\,m + \frac{1}{2}m + \frac{1}{4} - m\,e\,e - e\,e + e^4$ to which adde the former square of $\frac{1}{2}n + e$,

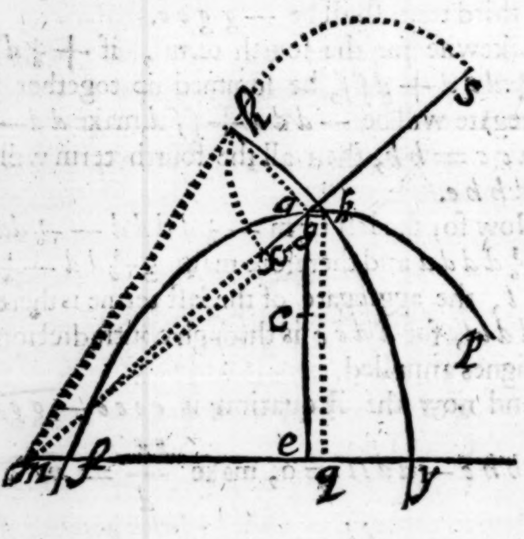
And then the whole is

$+\frac{1}{4}m\,m + \frac{1}{2}m + \frac{1}{4} - m\,e\,e - e\,e + e\,e\,e\,e + \frac{1}{4}n\,n + n\,e + e\,e$, That is,

$\frac{1}{4}m\,m + \frac{1}{2}m + \frac{1}{4} - m\,e\,e + e^4 + \frac{1}{4}n\,n + n\,e$ equall to the square of Radius $m\,k$.

Again, because $m\,e = \frac{1}{2}n$, the square of it is $\frac{1}{4}n\,n$, And because $a\,e = \frac{1}{2}m + \frac{1}{2}$, the square of that is $\frac{1}{4}m\,m + \frac{1}{2}m + \frac{1}{4}$, which added together make $\frac{1}{4}n\,n + \frac{1}{4}m\,m + \frac{1}{2}m + \frac{1}{4}$, for the square of $m\,a$, to which adde the square of $a\,h$, that is p (for $a\,x = p$, and $a\,s = 1$, and consequently by *Eucl.* 6. 13. the square of $a\,h$ is equal to

Or $eeee = mee - ne + p$, which is altogether the ſame with that in the former Chapter, and the working of it is there ſhewed. Except only becauſe there the quantity f is ſigned $-$, and here the like quantity p is ſigned $+$, I ſhall (although this caſe only is demonſtrated in *Des Cartes*) here demonſtrate it thus.



Deſcribe the *Parabola* $f\,a\,p$, according to the parameter d, (that is $a\,s$) & let $a\,e$ be the axis, & make $a\,c = \frac{1}{2}d$, $c\,e = \frac{1}{2}m$, and at right angles at e make $m\,e = \frac{1}{2}n$, and draw the line $m\,a\,s$, making $a\,s = d$, and $a\,x = p$, and upon $x\,s$ as a diameter deſcribe the ſemicircle $x\,h\,s$, and from a to h raiſe

raise the perpendicular $a\,h$, cutting the Circle in h, and with radius $m\,h$ describe the arch $h\,k\,y$, cutting the Section in k, and from k let fall a perpendicular to $m\,e$ produced in q, and draw the lines $m\,k$ and $m\,h$.

DEMONSTRATION.

To prove $g\,k = e$

Suppose it done, and because $k\,g = q\,e = e$, and $m\,e = \frac{1}{2}n$, therefore $m\,q = \frac{1}{2}n + e$, and the square of it is $\frac{1}{4}nn + ne + ee$.

And because $a\,e = \frac{1}{2}d + \frac{1}{2}m$, and $g\,k = e$, & because of the *Parabola* $g\,a = ee$, therefore also $e\,g$ or $q\,k = \frac{1}{2}m + \frac{1}{2}d - ee$, & the square thereof is $+\frac{1}{4}mm + \frac{1}{2}md + \frac{1}{4}dd - mee - dee + eeee$. That is (because d is equall to Unity) $+\frac{1}{4}mm + \frac{1}{2}m + \frac{1}{4} - mee - ee + e^4$ to which adde the former square of $\frac{1}{2}n + e$,

And then the whole is

$+\frac{1}{4}mm + \frac{1}{2}m + \frac{1}{4} - mee - ee + eeee + \frac{1}{4}nn + ne + ee$, That is,

$\frac{1}{4}mm + \frac{1}{2}m + \frac{1}{4} - mee + e^4 + \frac{1}{4}nn + ne$ equall to the square of Radius $m\,k$.

Again, because $m\,e = \frac{1}{2}n$, the square of it is $\frac{1}{4}nn$, And because $a\,e = \frac{1}{2}m + \frac{1}{2}$, the square of that is $\frac{1}{4}mm + \frac{1}{2}m + \frac{1}{4}$, which added together make $\frac{1}{4}nn + \frac{1}{4}mm + \frac{1}{2}m + \frac{1}{4}$, for the square of $m\,a$, to which adde the square of $a\,h$, that is p (for $a\,x = p$, and $a\,s = 1$, and consequently by *Eucl.* 6. 13. the square of $a\,h$ is equal

to

to p) the aggregate is $\frac{1}{4}nn + \frac{1}{4}mm + \frac{1}{2}m + \frac{1}{4} + p$. equall to the ſquare of Radius mh, but $mh = mk$. And therefore this aggregate is equall to the former aggregate & one may ſee that theſe quantities $+\frac{1}{4}mm + \frac{1}{4}nn + \frac{1}{4} + \frac{1}{2}m$. are comune to both; And therefore the reſiduals are equall.

Namely, $+eeee - mee + ne = +p$,
That is in *Des Cartes* his form.

$$eeee = +mee - ne + p,$$

Which was the æquation to be reſolved, and therefore $gk = e$, which was to be proved. And therefore if gk be added to $\frac{1}{2}d$, the ſum is equall to a the queſititious root of the firſt æquation, and is equall to bo. So that the point o which was firſt ſought, is hereby found; and the Probleme ſatisfied; which was to be done.

ADVERTISEMENT.

Now that the different ways of writing equations may cauſe no confuſion; let it be ſuppoſed to be written ever thus. eeee, mee, ne, p = o. *the ſignes* + *and* — *to be ſupplyed as the occaſion requires.* Then,

1 If it be $-m$, the center of the deſired Circle is within the Section, or at leaſt below the Vertex.

2 If it be $+m$, the ſaid center is above the vertex, and $\frac{1}{2}m$ is applied upon the Axis produced, from the point c, which is always in the axis within

within the Section, distant from the vertex *a* by halfe the parameter; and therefore in this case the line *m* cannot be lesse then the parameter *d*, otherwise ths point *d*, would still fall within the Section.

3 If it be + *n*, those perpendiculars (let fall from the severall intersections of the *Parabola* and Circle to the Axis) which are on the same side with the center are the false roots, and the other the true roots, but if it be — *n*, then just contrary.

4 Lastly, if it be — *p*, then one auxiliary circle will serve, as here it doth, but if it be + *p*, then there must be another also, the describing of both which is shewed in the former Chapter.

Probl. 2.

Upon a line given as a Base to describe an Isosceles triangle, so that an inward parallel Base may cut off two segments of the sides betwixt the bases, so that either segment may be equal to the inwad base, the perpendicular from the vertex to the said inward Base being also by supposition given.

Let there be given the line *c d*, and the line *b g*.

And let it be required upon *c d*, as a base, to describe the *Isosceles b c d*, so as the line *b g*, falling at right angles with *f e*, equidistant to *c d*, the lines *d f*, *e f*, and *e c*, may be equall each to other.

Put-

Put $gr = e$, and $fe = a$,
And $cd = b$ and $bg = d$,
Then $d\,'$, $d + e''$, $a'\,b''$,
That is $da + ea = db$.
Euclid. 6. 16. which is the ſame æquation as was firſt in the former Probleme; and therefore if there be in the roome of $da + ea = db$ ſubſtituted another equation like that in the former, ſuch is the æquation.

$a^4 - 2daaa + ffaa + 2dcca - ddcc = 0$.

And that purged from the ſecond term, as before, there ariſeth a ſecond equation.

$$+ eeee - mee + ne - p = o$$

And laſtly, if d be a parameter, according to which a *Parabola* is deſcribed, the root e, and conſequently the root a, may be found as in the former.

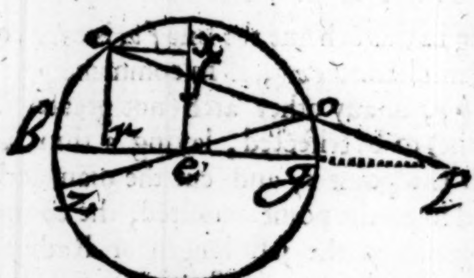
And thus having ſhewed the method general for all Æquations which attain but 3 or 4 dimenſions, and exemplified it by Problemes which lead to ſuch æquations; I now ſay that was the end of my preſent buſineſſe. And if any ſtill deſire a longer reach, I referre him to *Des Cartes*, who hath proceeded to æquations of 5. and 6. dimenſions; by which foure Mean proportionals, and quinquiſection of Angles, and other ſurſolid Problemes may be found and effected.

Note.

Note 1.

The firſt Probleme of this Chapter, as it is more compoſed then triſection, ſo it comprehends it; as may be ſeen by (not only *Pappus* and others who applied it to that end but) the following Example. In which, let there be an arch of a circle, namely *b c*, and let it be required to divide the arch *b c*, into three equal parts, or (which is as good) to find the third part of the arch *b c*.

Find out the center *e*, and deſcribe the circle, and draw the Diameter *b g*, and produce it to *p*, or further, as is need, and make *c r* the right ſine



of the arch *b c*, and from the center *e*; draw *e x* parallel to *c r*, and complete the rectangle *c x e r*, and by the firſt Probleme, draw *c p* to cut *e x*, in *f*, ſo as *f p*, may be equal to *b g*, then I ſay that *g o* is equal to a third part of *b c*.

From *o* through *e*, draw *o e z*.

Now becauſe *e o* is Radius, *f p* the diameter,

and the angle *f e p* a right angle, therefore the lines *f o*, *p o*, *e o*, are severally equal.

And the angles *z e b* = *g e o*. and *p e o* = *e p o*.

And also *f o e* = 2 *p e o*. *Euclid*. 1. 32.

Therefore also *f o e* = 2 *z e b*.

But *f o e*, that is *c o z*, being in the peripherie, is measured by half the arch *c z*, *Euclid*. 3. 20. wherefore *b z* which is the measure of half the angle *c o z*, is a fourth part of the whole arch *c z*; and consequently a third part of *b c*, and therefore *g o* which is equal to *b z*, is also equal to a third of *b c*, which was to be proved.

NOTE 2. *Mechanically.*]

Seeing in this Scheme the line *f o*, is ever equal to the Semidiameter *e g*, if in commune practise there is *b c*, or any other arch (not greater then a quadrant) to be trisected, laying a thin Ruler to touch the point *c*, and cut the diameter *b g*, produced in *p*, the point required, the compasse being opened to the just lengrh of Radius *e g*, setting one foot in *e x*, and shifting the Ruler till the other foot fall in the peripherie at *o*, the point *o*, shall always be distant from *g*, by a third of *b c*, the doing of which (although it must not be called Mathematical, yet) is very neer as easie, and as free from erring as from a point given to a point given to draw a streight line; or upon a center given with a distance given to describe a circle; & from a given point in it to set off an arch equal

equal to an arch given : And therefore I recommend this as the most simple and short and safe way for Mechanick use.

NOTE. 3

If the arch to be trisected be greater then a quadrant, then trisect the complement thereof to a semicircle ; and the third of this complement taken from 60 degrees (which is always a given arch) leaves the third of the arch required.

Inscription of Chords in a Circle, and making æquicrurall triangles whose angle at the base shall be to the angle at the vertex in any given proportion, are the same thing : for to inscribe a figure of 3, 4, 5, 6, 7, 8, 9, 10, 11, sides &c, findes such triangles whose said angles shall be as $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$, $\frac{5}{2}$, $\frac{6}{2}$, $\frac{7}{2}$, $\frac{8}{2}$, $\frac{9}{2}$, $\frac{10}{2}$, &c, as is easie to be seen by the operation.

Quadrature of the Circle is that in which (as yet) onely *Archimedes* hath laboured with any successe ; he having demonstrated that the Circumference is to the Diameter lesser then as 22, to 7, and greater then 21 $\frac{70}{71}$, to 7, within which strict limits , a French man many years since found it to be in whole Numbers. thus,

As the whole circumference, is to the perimeter of the inscribed Square : so is 10, to 9, that is. Quadrant′. Subtense ″, 10′. 9″. which is easie from practise ; and may be proved. thus,

Put the Diameter $=$ 7, the halfe $=$ 3 $\frac{1}{2}$, of

which the ſquar is $= \frac{49}{4}$, and doubled is $\frac{49}{2}$, whoſe ſquare root is the ſide of the inſcribed ſquare. The whole perimeter therefore is $4 \sqrt{\frac{49}{2}}$, and the whole circumference is found by this Analogiſme.

$$9'.\ 10''.\ 4\sqrt{\tfrac{49}{2}}'.\ \tfrac{40}{9}\sqrt{\tfrac{49}{2}}''.$$

It reſts to be proved that $\frac{40}{9} \sqrt{\frac{49}{2}}$ is greater then $21\frac{70}{71}$, and leſſer then 22.

Now 4,949, is ſomthing leſſe then $\sqrt{\frac{49}{2}}$ which multiplied by 40, and divided by 9, the quotient it 21,995, which yet is greater then $21\frac{70}{71}$. for $\frac{995}{1000} > \frac{70}{71}$.

Again 4, 950. is ſomthing greater then $\sqrt{\frac{49}{2}}$, which multiplied by 40, and divided by 9, as before, the quotient is 22, ſo that $\frac{40}{9} \sqrt{\frac{49}{2}} < 22$. and $\frac{40}{9} \sqrt{\frac{49}{2}} > 21\frac{70}{71}$. which was to be proved

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CHAP. XVI.

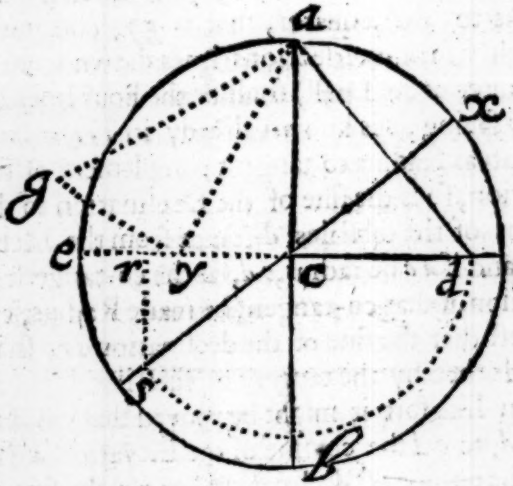
Of Dialling.

Probl. 1.

Upon any declining Plane, to finde the height of the Stile, and place of the Subſtile, the Declination being firſt known.

THe lines *ab* and *ed*, drawn to cut at right angles in *c*, and the angle *cad*, made the complement of Horizontal elevation: upon the center *c*, and ſpace *ca*, deſcribe the quadrant *aq*, and

and make *q x* the declination, and draw the declining Plane *x c s*, and on the center *c*, and ſpace *c d*, deſcribe the arch *d s*, cutting the decliner in *s*, from whence let fall to *d c* produced, a perpendicular *s r*. And make *c y* = *r s*, and draw the



line *a y* for the ſubſtile. Secondly make *y g* = *cr*, and the angle *a y g* a right angle, and by *g*, draw *a g* for the Stile.

Then if from the point *y*, to the ſtile, be let fall a perpendicular: and a line equal to it ſet off from *y*, towards *a*, it ſhall be the ſemidiameter of the æquator. Which æqaator being deſcribed a line drawn from the center thereof & produced to the point where the tangent *g y* would interſect *a c*, produced

produced ſhal cut it into two equal parts, whereof that ſemicircle which is neerer to the line *y g* divided into 12 equal parts beginning at the ſaid point of interſection of *g y*, & *a c*, produced & reckoning both ways as the Plane *y q*, ſhall admit, a Ruler layd from the center of the æquator to every one of thoſe diviſions, ſhall interſect the tangent to the æquator, that is *g y*, continued through which interſections lines drawn from *a*, (the center of the Diall) ſhall be the hour lines.

For ſeeing it is known already in *Trigonometrie* that, as Radius to tangent complement of the Elevation, ſo is the ſine of the Declination to the tangent of the ſubſtiles diſtance from the Meridian, and if *a c* be radius, *c d*, is the co-tangent of Elevation, if that co-tangent be made Radius, it is manifeſt that the ſine of the declination *s r*, ſhall be made thereby the tangent of ſubſtilar diſtance, *y c*. In like ſort it might be proved that, as Radius *c d*, to *c d* the co-ſine of the Elevation *a d c*, ſo *r c* the co-ſine of declination, to *r c* the ſine of *y a g*.

Probl. 2.

Of Declining Recliners.

To finde the Meridian of the Place, the Meridian of the Plane, and height of the Stile.

Let the parallel lines *q s*, *l g*, repreſent the baſe of the declining reclining plane; and make the angle *g x n* equal to the reclination,

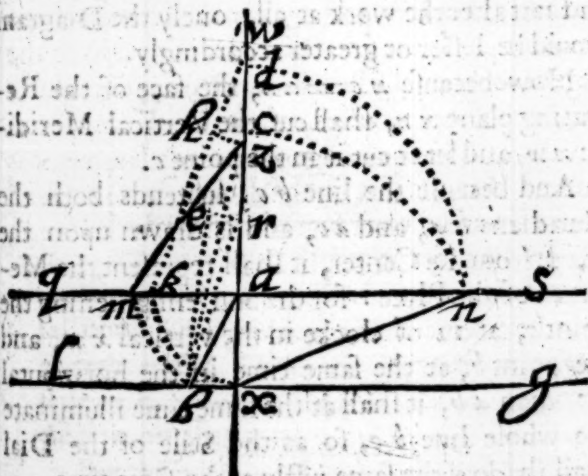
And

And let *q s*, and *x w*, cut at right angle in *a*.
Make the angle *x a b*, equal to the declination.
And draw the lines *x n*, and *a b*.
And making *x c* = *x n*, draw *c b*, for the Meridian of the place; or hour of 12.

Then make *a m* = *a b*, and the angle *a m z*, equal to the latitude, and draw the line *m z*.

Make *a k* = *a x*, and *a d* = *a n*.

Draw the line *k d*, and (where is need) produce it.



From *z*, let fall a perpendicular to *k d*, in *h*, and from *b*, (the center of the Dial) to the vertical Meridian *x w*, apply *b r* = *k b*, for the Substile.

Lastly unto the line *b r*, in the point *r*, raise a perpendicular *r e*, = *z h*, and draw a line from *b* to *e*, for the Stile.

So the line *r e*, produced both ways, shall be a tangent to the Æquator, whose semidiameter shall be a line let fall from the point *r*, perpendicular to the line *b e*. And the rest of the Dial may be finished like a Vertical decliner, as in *Prob.* 1

DEMONSTRATION.

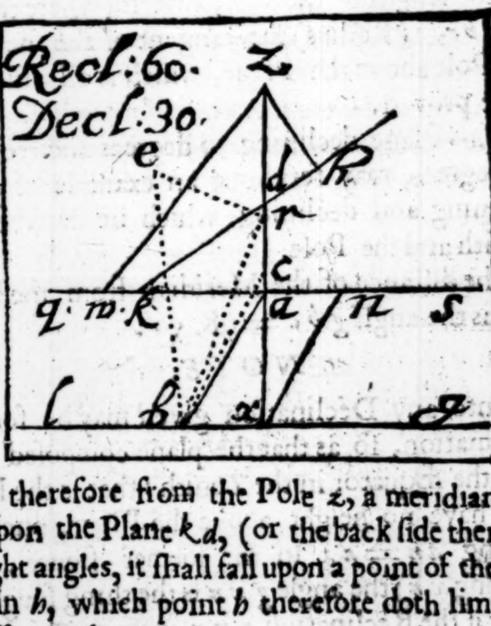
First the Center *b*, is chosen at pleasure, in any place of the Horizontal Meridian *a b*, for the Paralleis *q s*, *l g*, might be put neerer or further off and not alter the work at all, onely the Diagram would be lesser or greater accordingly.

Now because *x c* = *x n*, the face of the Reclining plane *x n*, shall cut the Vertical Meridian *x w*, and let it cut it in the point *c*.

And because the line *b e*, subtends both the Meridians *b a*, and *a c*, and is drawn upon the Plane from the Center, it shall represent the Meridian of the Place: for the Sun enlightening the point *c*, at 12 of clocke in the vertical *x w*, and the point *b*, at the same time in the horizontal meridian *a b*, it shall at the same time illuminate the whole line *b c*, so as the Stile of the Dial shall shadow the same justly at the same time.

Again, because the triangles *a n x*, *a k d*, are æquiangular and equal, if the point *w*, be the Zenith, the point *x* supposed to be laid upon the point *k*, the lines *k d* = *x n*, and the angle *a k d* equal to the reclination, *k d* shall then truly represent the reclining Plane. More

Moreover *am* being equal to *ab* by constructi-on, and the angle *amz* equal to the latitude or elevation, the point *z* represents a Pole of the Æquator, because the Axis *mz*, and the Meridian *az* meet there.



If therefore from the Pole *z*, a meridian *zh* fall upon the Plane *kd*, (or the back side thereof) at right angles, it shall fall upon a point of the substile in *h*, which point *h* therefore doth limit the substile on that part.

But the horizontal line in which *b*, the center of the dial is taken must limit it on the other part, to wit in the point *x*, or *k*, so that *hk* is the just length of the substile upon the Plane.

And because the substile must both passe through the Center *b*, and incline to the vertical *xw* (to which

which the plane it self inclines) the line $b\,r = k\,h$ being so placed is the substile.

Lastly, because $z\,h$ is the neerest distance betwixt the Pole and the Plane, $e\,r$ being equal thereunto, and perpendicular to the Substile, it shal be the length of the Stile, that is (where the Substile $b\,r$, is Radius) the tangent of the height of the Pole above the Plane, which is all which was to be proved.

This Plane declining 30 degrees and reclining 20 degrees, may serve for an example of planes reclining and declining which lie between the Zenith and the Pole.

The distance of the Meridian from the Horizon is the angle $g\,b\,c = 78.50'$.

NOTE.

Unto any Declination given may be found a Reclination, so as that the plane composed shall have the Æquator in the Zenith: that is the Poles shall have no height above the Plane, onely by making $a\,n = a\,z$ in the former figure, and drawing $x\,n$; the angle $g\,x\,n$ is the thing sought.

Or if the Reclination $g\,x\,n$, were given, a Declination might be found to doe the same, onely by making still $a\,z = a\,n$; and the angle $a\,z\,m$ equal to the height of the Æquator above the Horizon; and $a\,b = a\,m$; and drawing the line $a\,b$, the angle $x\,a\,b$, shall be the declination sought for.

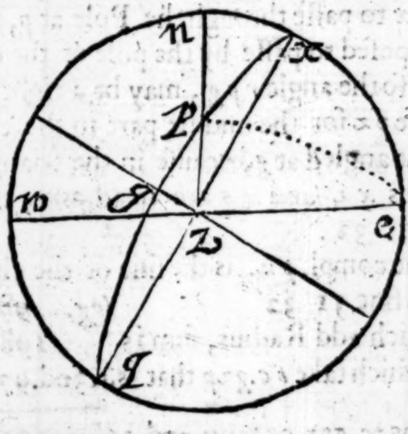
And those Planes so made are called meridi-

onal

onal Planes, becauſe, they are parallel to the Axis of the Æquator. The Dial upon any of them is like the Polar Plane, having the ſame reaſon for the paralleliſme of the hours; but becauſe the Subſtile may chance to fall neither upon our 12, nor 6, but any where elſe according to ſeveral declinations, the place of that muſt be found.

Which is done by finding a certain angle comprehended betwixt the Meridian of the place and the Meridian of the Plane, which angle is commonly called *the angle of inclination of Meridians*, and may be found as follows.

Deſcribe the following Circle *w n e s* for the Horizon, and draw *n s* for the Meridian, and *w e*,



for the Prime Vertical, *q g x*, for the declining Plane; *p r*, the Meridian upon it, and *p n* the elevation of the Pole above the Horizon. Then in the

triangle

triangle *pgz* right angled at *g*, it is evident by the first Axiome of Spherical triangles, that if you work by the artificial sines & tangents, & also chuse *pz* for the middle part, the Æquation will be this.

s.c.pz + Radius = *tc.gzp* + *tc.gpz*.

And therefore adding the sine of the elevation to Radius, and from the aggregate subtracting the tangent complement of the declination, the remaine will be the tangent complement of the angle of inclination of Meridians, which angle sought is *zpr* for *gpr* is 90 d.

Example.

Let the Declination *wzq* be 60 degrees.

The Reclination *zg*, so much as may cause the Plane *qgx* to passe through the Pole at *p*, and let *pr* be supposed to passe by the pole of the circle *qgx*, that so the angle *rpg*, may be a right angle: and choose *pz* for the middle part in the triangle *bgz*, right angled at *g*, because in the triangle *gxz*, the sides *xz* and *xg* are quadrants, then in latitude 51. 32′.

The sine compl. *pz*, is the sine of the elvation,

That is sine 51 32′	*log.*	9893745
To which add Radius, sum is		19893745
From which take *tc.gzp* that is *t*. 30d.		9761439
Remains *tc.gpz* or tang. 53d. 36′. =		10132306

The angle *fpz*, therefore is 53. 36′. Which is the angle of inclination of meridians sought for, which being divided by 15 (the number of deg. of

the

the Æquator accompted for every hour) the quotient is $3\frac{57}{100}$, or rather $3\frac{172}{300}$. which ſhews the hour from Noon over which the ſtile muſt hang that is in the after noon $3\frac{172}{300}$ of clock if the declination be weſterly, or $8\frac{128}{300}$. if eaſterly, this laſt the morning hour, the other afternoon.

Now this fraction being rectified (which every man that hath any ſkil in Arithmetique knows how to doe) and the intire hours of 8 and 9, or 4 and 3 being aſſigned, the reſt may be found by tangents of arches encreaſing ſtil from the ſubſtile by the quantity of 15 d. for every hour. And ſo the Dial may be made on paper.

But to place it right after it is made, the angle comprehended betwixt the ſubſtile and the horizontal line which is here the line *l g* upon the Reclining Plane, muſt alſo be found out: And may thus (working ſtil by Logarithmes) *from the ſine of the latitude plus Radius, take the ſine complement of the Reclination, there ſhall remain the ſine of an angle, which angle is the true diſtance of the ſubſtile from the Horizon.* And muſt be ſet off from the horizontal line at the center of the dial weſtward if the declination (as here it is) be eaſtward: Or elſe eaſtward if the declination be weſtward. And ſo the dial ſhall be rightly made and ſituate.

Now though all this be moſt eaſie to all that know how to uſe the Logarithmes; yet that this may not depend thereon, the ſame things may be found

found out Geometrically by describing any Circle at pleasure.

For first, *Cotangent Declination′. Radius′′. sine latitude′. tangent Inclination of Meridians′′.*

Secondly, *Cosine Reclination′, sine Latitude′′; Radius′. sine distance of substile from Horizon′′.*

Which is enough, any circle will afford these naturally; for such as affect not the artificial, and the former Scheme will demonstrate this easily.

Prob. 3.

To find the same in those called South Declining Inclining Planes.

Put the parallel lines *p*, *q*, for the horizontal Base of the inclining Plane.

The Declination, angle *a b c* = 60 d.

The Inclination *p c d* = 58 d.

Make *y z* perpendicular to the line *q* in *b*.

Make also *c e* = *c d*.

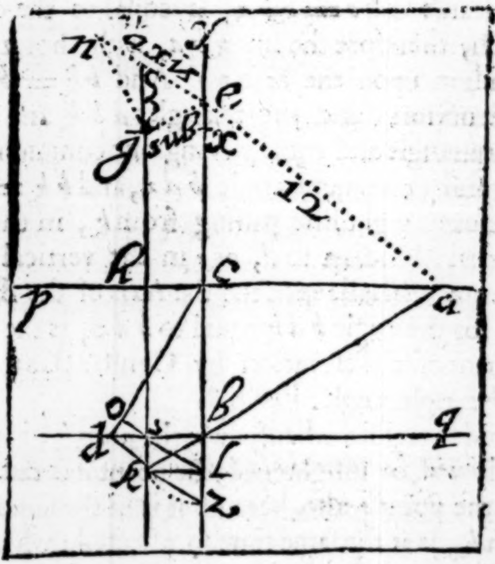
And *b d z* equal to the Complement of the elevation.

Joyne *b* and *z*. And make *b k* = *b z*.

Through *k* (the line *a b* being first produced so far) draw *k u* parallel to *z y*, cutting the line *q d* in *s*.

And

And make $bl=sk$.



From l let fall a perpendicular to $c\,d$ to wit $l\,o$.

Make $hg=co$.

Then draw eg for the substile. And,

Make $g\,n=l\,o$, and at right angles with eg.

And draw $e\,n$ for the Stile.

And $e\,a$ for the houre of 12,

So the Dial may be finished like a Declining Verticall.

DE.

DEMONSTRATION.

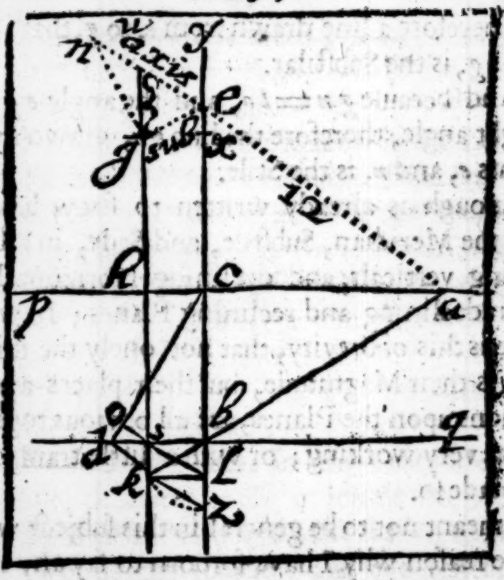
Becauſe $abc = kbz$, is equal to the declination, therefore the line kba, is the horizontal Meridian upon the baſe pq. And $bk = bz$ by conſtruction, and the triangles dbk and dbz are equiangle and equal, having one commune ſide db; and a commune angle bdz, and $bk = bz$, therefore a right line paſſing from k, in the horizontal Meridian to d, or e in the vertical Meridian zy, ſhall repreſent the Axis of the Æquator; for the angle bdk equal to bdz, is the complement of the elevation by Conſtruction; and dbk a right angle.

And therefore whenſoever the poin k, is either ſhadowed, or inlightened, the point a is the ſame; and the point e alſo, becauſe it is in the ſame Axis with k, is at the ſame time ſo affected; wherefore the center of the dial being at e, a line drawn from thence upon the Plane to a, ſhall be the hour of 12.

And becauſe the Hypothenuſa dk, or ek, is the Axis paſſing from e, in the inclining Plane by k, in the horizontal Meridian; and the point k being in the line $ksbg$, a perpendicular let fall from thence to the Plane ſhall fall in the ſame line $ksbg$.

Make $ex = bg = co$. Then a perpendicular from l to x, is the ſame with that from l to o, namely the line lo.

And



And because $b\,l = s\,k$. and $c\,x = b\,g$, therefore a perpendicular let fall from k, to g, shall be equal to $l\,x$, or $l\,o$.

And because k (as hath been shewed) is a point in the Axis, and g, a point in the Dial Plane, the perpendicular $k\,g$, or $l\,o$, shall be the height, or (as some call it) the length of the Stile; or more properly (making $e\,g$, radius) the tangent of that height whereby the Pole is elevated above the Plane.

And the point g, where it cuts the Dial plane at right angles, shall be a point in the substile.

But e, or d, is the Center of the Dial, as hath been shewed already.

There-

DEMONSTRATION.

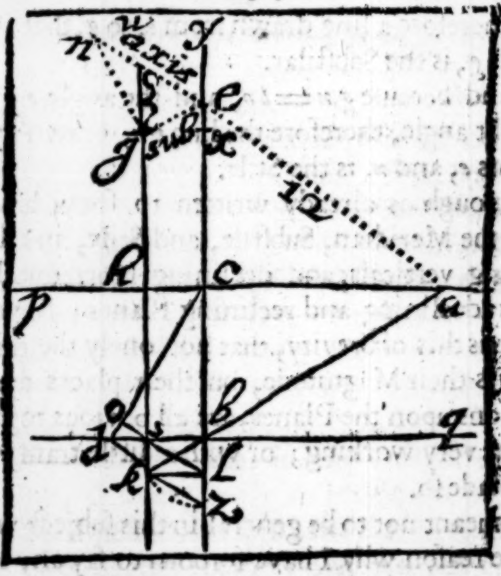
Becauſe *a b c* = *k b z*, is equal to the decli-nation, therefore the line *k b a*, is the horizontal Meridian upon the baſe *p q*. And *b k* = *b z* by conſtruction, and the triangles *d b k* and *d b z* are equiangle and equal, having one commune ſide *d b*; and a commune angle *b d z*, and *b k* = *b z*, therefore a right line paſſing from *k*, in the horizontal Meridian to *d*, or *e* in the vertical Meridian *z y*, ſhall repreſent the Axis of the Æquator; for the angle *b d k* equal to *b d z*, is the complement of the elevation by Conſtruction; and *d b k* a right angle.

And therefore whenſoever the poin *k*, is either ſhadowed, or inlightened, the point *a* is the ſame; and the point *e* alſo, becauſe it is in the ſame Axis with *k*, is at the ſame time ſo affected; wherefore the center of the dial being at *e*, a line drawn from thence upon the Plane to *a*, ſhall be the hour of 12.

And becauſe the Hypothenuſa *d k*, or *e k*, is the Axis paſſing from *e*, in the inclining Plane by *k*, in the horizontal Meridian; and the point *k* being in the line *k s b g*, a perpendicular let fall from thence to the Plane ſhall fall in the ſame line *k s b g*.

Make *e x* = *b g* = *c o*. Then a perpendicular from *l* to *x*, is the ſame with that from *l* to *o*, namely the line *l o*.

And



And because $b\,l = s\,k$. and $c\,x = h\,g$, therefore a perpendicular let fall from k, to g, shall be equal to $l\,x$, or $l\,o$.

And because k (as hath been shewed) is a point in the Axis, and g, a point in the Dial Plane, the perpendicular $k\,g$, or $l\,o$, shall be the height, or (as some call it) the length of the Stile; or more properly (making $e\,g$, radius) the tangent of that height whereby the Pole is elevated above the Plane.

And the point g, where it cuts the Dial plane at right angles, shall be a point in the substile.

But o, or d, is the Center of the Dial, as hath been shewed already.

There-

Therefore a line drawn from *e*, to *g*, that is the line *e g*, is the Subſtilar.

And becauſe $g\ n = l\ o$, and the angle *e g n*, is a right angle, therefore the line *e n*, drawn by the points *e*, and *n*, is the Stile.

Enough is already written to ſhew how to find the Meridian, Subſtile, and Stile, in all declining verticals, and declining Horizontals, I mean declining and reclining Planes; In which there is this of brevity, that not onely the things, that is their Magnitude, but their places and Situations upon the Planes, are all obvious together in the very working; or with a little tranſpoſition made ſo.

I meant not to be general in this ſubject which is the reaſon why I have forborn to ſay any thing of Horizontals, Prime Verticals, Equinoctials, and Polar Dials. Yet becauſe the book ſhall not be rendred (to ſome perſons) uſeleſſe for want of theſe; at the end hereof I purpoſe to append a Table (out of *Kercherus*) by which the Horizontal and Prime Verticals all over *Europe* eſpecially, may be made by the quantities of their Arches ſet down in the table and to be ſet off from the Meridian of the plane upon the Dial Plane, and may be meaſured upon any Circle there deſcribed at pleaſure.

Æquinoctial Planes (I mean ſuch as are ſo denominated from the planes not the Poles) are ſuch as have one of the Poles for Zenith; upon

these, a circle divided into 24 equal parts gives the hours; and a pin perpendicular to the center, (of any length) is the Stile.

Polar Planes have the Æquator in the Zenith; where these are proper Horizons, the Substile and Meridian of the Place are all one; the hours parallel to it; their distances from it measured by tangents of 15, 30, 45, 60, 75, 90, &c, degrees according to any Radius; provided that Radius be the same with the height of the stile; which is a pin set upright in the center of the Circle to which the tangents belong.

East and *West Planes* in any latitude are of this kinde, differing onely in longitude, 90. degrees; by which it comes to passe that in these the houre of 6. is the substile, and the rest of the worke (leaving out unnecessary hours) the same as in the former.

To place these right upon the plane draw a line parallel to the horizon; and in any convenient place from the horizontal line in an arch of some circle set off the latitude; included between the horizontal line & another, that other line, cut by a third line at right angles, the third line shall be the Æquator, the second the substile, the rest like that before. *Prob.* 1.

Probl. 4.

To draw any Verticall Dial by help of an Horizontal dial, without any Æquator.

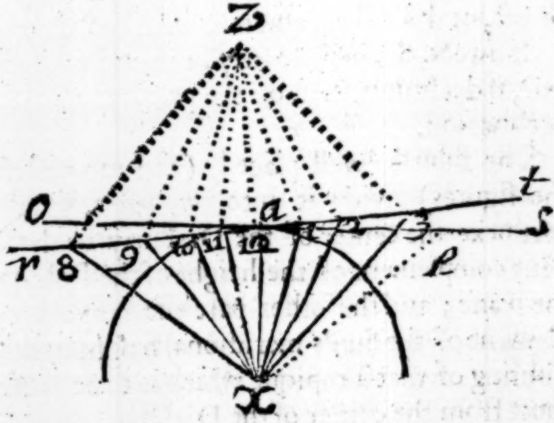
Making the Center *x*, and at any distance de-

ſcribe a circle, on which (having made *x a* the Meridian) ſet off the horizontal Arches proper to the latitude (taken out of the table hereafter following, or any other way;) from the Meridian making thereby markes in the circle, by which, and the center *x* draw the horizontal lines *x* 1. *x* 2. *x* 3. &c. on both ſides the Meridian to cut the declining plane (which in this example is the line *r t*, declining from the Prime Vertical *o s*, as-much as is the angle *o a r* that is almoſt ſeven degrees) in the points 1, 2, 3, &c, on the one ſide; and 8, 9, 10, and 11, or as many of them as the plane will receive on the other ſide of the Meridian: to which points lines drawn from the center of the dial, that is *z* ſhall be the hour lines, the angle *a x p*, is the elevation.

The ſaid center *z*, is found out by making, *a z* = *a p*, the ſaid *a p* being always the ſine of the latitude, where the line *a x* is the ſine of the complement thereof, that is, having made *x a* the meridian, and *o s*, the prime vertical, and the angle *a x p* equal to the latitude, the point *p*, and the line *a p*, are thereby given, then having made *z a*, perpendicular to *r t*, make *z a* equal to *a p*.

Now to find the ſtile and ſubſtile, it is already ſhewed at firſt in the vertical decliner.

Concerning *Azimuths*, *Almicanters*, and other ſuch things, I ſhall not ſay much, becauſe where there needs moſt Art to deſcribe them, there



there they are most uselesse; serving but to make a Dial more blind, which looked too much a squint before. And although the Dialler have all prompt in his head; yet very seldom doth his hand so concurre herein as to inscribe these things in their right places, especially in oblique Planes.

Before I medle at all with these, it will be necessary to proportion the Perpendicular Stile to the Plane.

CHAP. XVII.

Of the Perpendicular Stile.

THis must be perpendicular to the substile, and the top thereof determined in the Stile, or axis.

If the Plane be ſmal, conſider whether it be direct, or declining, and much declining.

If direct, the ſubſtile may be placed in the midſt, if declining, then on the part oppoſite to the declination.

The ſubſtile well placed, (and roome left for the figures) divide it into two parts, ſo as that part next the center of the Dial may be the tangent complement of the height of the Pole above the plane; and the other part the tangent complement of the Sun's meridional height in the beginning of that Tropique which is to be more remote from the center of the Dial.

And the Radius proper to theſe tangents ſhall be the perpendicular ſtile, to be placed in the point of Diviſion in the ſubſtile, perpendicular thereunto.

Of the Signes, or Parallels.

A Signe is a twelfth part of the Ecliptique, and containes therefore 30 degrees.

A Parallel, according to the vulgar ſenſe, is the Suns diurnal Motion day by day: And becauſe there are 47 degrees from Tropique to Tropique, there may be ſo many Parallels, that is, circles which the Sun deſcribes every 24 houres ſuppoſed Parallel to the Æquator though not exactly ſo; and although there are 47 of theſe yet in our latitude of 51. 32′. we accompt but 9. *viz.* thoſe which are the day from Sun to Sun when it is 8,

9, 10, 11, 12, 13, 14, 15, or 16. hours long. The Description of these parallels and of the signes is made the same way: onely due respect must be had to the quantity of the Suns declination, for in all direct horizontals, the perpendicular stile being Radius, the tangent complement of the Suns height, in any signe or parallel, at any hour of of the day, set off from the foot of the said stile, and extended to the hour line, gives a marke, by which the parallel of that day shall passe. So that this Worke repeated so often as the number of parallels to be inscribed, and the hour lines require, shall give respective points enough in each hour to draw each parallel by.

Example.

In the latitude 51. 32′. the Sun being in *Pisces* (the beginning thereof) the degrees of the Suns height above the horizon at every hour being as followeth, that is, 25. 37′. at one of clock, 21. 49′. at two; 15. 57′. at three; 8. 32′. at foure, and the same for eight, nine, ten, and eleven respectively, if the perpendicular stile being Radius, the tangents of the complements of 25. 37′. 21. 49′. 15. 57′. 8. 32′. be applyed from the foot of the stile to the respective hour, that is, the cotangent of 25. 37′. from the foot of the stile to the hours of 1. and 11. and so the others, they shall give points in every hour-line one, by which a line being drawn with an even hand shall be the

Parallel at the beginning of *Pisces*. And the like of all the rest.

And therefore generally in verticals, as also in all recliners that is to say upon all planes whatsoever, draw a horizontal dial proper to the plane, and inscribe the signes or parallels upon it, by setting off from the foot of the perpendicular stile, the tangents complements of the Suns height at every hour in the beginning of every such signe, above that plane taken as an horizon, the perpendicular stile being ever Radius; and at the end of these tangents so set off, upon every respective hour-line, will be a point, by which points, lines drawn with an even hand, shall give the parallels desired. This horizontal Dial being drawn in obscure lines, the Dial for the plane may be drawn afterwards. The Parallels serving which were drawn before.

Example.

Suppose (as M. *Wells* doth *pag.* 185) a plane declining 30 degrees, and reclining 55 degrees; the height of the Pole above the plane 19 degrees 25 minutes; the Suns height at the beginning

	12h.	82d.	5'.
Of *Taurus* to be at the hours of	1	73	30
	2	60	3
	3	46	1
	4	31	53
	5	17	47

The

The tangents of the complements of 82, 5′. and 73, 30′. and 60, 3′. &c. ſet off from the foot of the perpendicular ſtile (the ſaid ſtile being the Radius to thoſe tangents) to the obſcure horizontal hours of 12, 1, 2, &c. give the true diſtances between the foot of the ſtile and thoſe auxiliary hours for the parallel of *Taurus*. And ſo the other Parallels may be found.

It is true, the height of the Sun at every hour of the day, at the beginning of every ſigne in any latitude is not eaſily found out without Trigonometrical Calculation by Logarithms of the ſines & Tangents, or by truſting to Tables already Calculated, if any happen to be done for that latitude already, the way of making a table ſhall be ſhewed towards the end.

Of the Vertical Circles.

Theſe are vulgarly called Azimuths; and are great Circles whoſe Poles lye in the horizon, and interſecting one another in the Zenith and Nadir of the Place.

The whole Horizon being divided into 32 parts equal, theſe circles ſhewing thoſe diviſions are called points of the Compaſſe, and marked *S. S b E. S S E.* &c. Every one diſtant from other by, 11 ¼ degrees.

But the better way of accompting them is 10, 20, 30, 40, 50, 60, 70, &c. degrees from the Meridian.

1 In

1 In all horizontal dials, the Perpendicular ſtile being choſen, making the foot thereof the center, at any convenient diſtance deſcribe a circle, and accompt from the meridian both ways arches equal to 10, 20, 30, &c. degrees, from which diviſions right lines drawn to the foot of the ſtile aforeſaid, ſhall repreſent thoſe Azimuths upon that dial.

2 Upon a Prime Vertical (or South) Dial, through the foot of the perpendicular ſtile draw a right line parallel to the horizon, and making the ſaid ſtile radius, upon the parallel line ſet off both ways from the Meridian tangents of 10, 20, 30, 40, &c. degrees, through which diviſions right lines drawn all at right angles with the parallel line ſhall be the Azimuths.

3 Upon any declining Vertical the ſame being done ſhall give the Azimuths of 10, 20, 30, &c. from the meridian of the plane, or from the Meridian of the place, juſt allowance being made for the diſtance of Meridians.

4 In South Declining Reclining Planes, the perpendicular ſtile being choſen, and made the Radius, the tangent complement of the Reclination applyed from the foot of the ſaid ſtile to the meridian of the place, ſhall determine the Zenith of the place, through which, and the foot of the ſtile, that is the Zenith of the plane, a right line drawn ſhall be a perpendicular to the horizontal line, which ſhall concurre with the æquator in the hour

hour of 6, and the therefore if from the foot of the stile upon the said perpendicular towards the North (for the former application is made towards the South) be set off the tangent of the reclination, a line drawn from the end thereof at right angles with it, shall be the horizontal line: upon which the tangents of 10, 20, 30, &c, (the secant of the reclination being now made Radius) set from the said right angle, lines drawn from them to the Zenith of the place shall be the Azimuths.

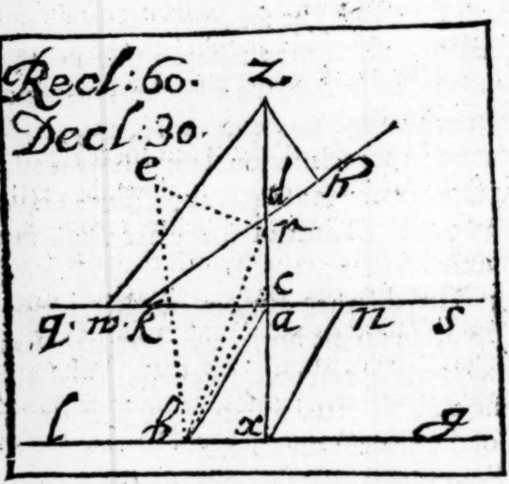
5 The distance betwixt the Meridians being known, upon the horizontal line, the Azimuths which were accompted from the meridian of the plane may be fitted for accompt from the meridian of the place with easie.

For example, let that distance be the tangent of 20 deg. then that Azimuth which is 10 from the one, is 10 from the other also, and that which is 30 on the same side of the substile, is 10 on the other side of the Meridian of the place, the like Method serves for any distance.

Note 1.

It may be noted, that although I have shewed the construction of a South reclining plane at the beginning hereof in a figure proper only to those planes which recline not further then the Pole, whereas in those that doe, and although there be some variation of the Scheme as you may see by com-

comparing this with the former (at the first beginning of this subject) for the point *h*, which



there fell on that side of the vertical meridian *z x* towards *q*, here falls on the other side towards *s*, likewise the hour of 12, that is *b c*, did there fall betwixt the axis and the substile, but falls here betwixt the substile and the horizontal meridian *b a*: yet this notwithstanding the construction is the very same in both.

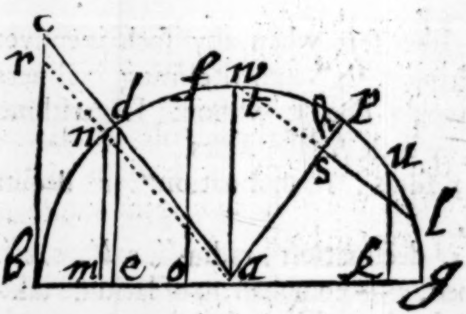
NOTE. 2.

It may be further Noted, that as the Reclination may increase, the points *n*, *c*, *r*, all approach still neerer to *a*, and when the reclination is 90 they

they are all coincident, and this vaniſheth into an horizontal dial whoſe ſubſtile will be *b a.*

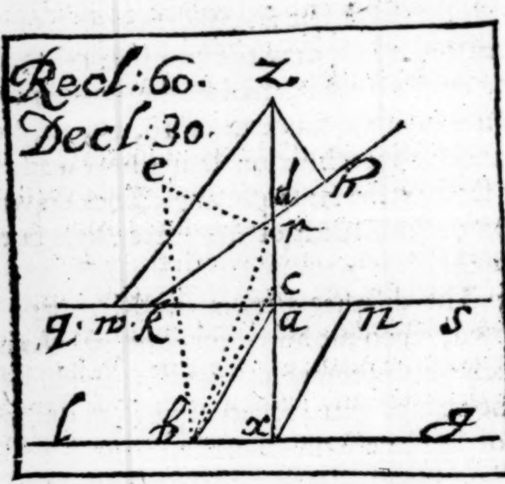
Alſo if the Declination be ſtill increaſed, at laſt the points *b* and *m*, will be coincident, and the dial plane will be parallel to the prime verti-cal, and the work a South dial whoſe ſubſtile is or may be *z a.*

But whether the declination increaſe or not, if the reclination amount to 70 d. or above, then the ſubſtilar line applyed (as before) from *b*, to the vertical *x z*, ſhall fall between the points *a* and *x*, upon the baſe of the plane. By which means the former method is rendered inſufficient, or (at leaſt) inconvenient in this caſe. Wherefore ha-ving deſcribed any ſemicircle on paper, and made therein the angle *d a b*, equal to the declination



(which let be 50 d.) and the angle *f a b* equal to the reclination (which let be 75) make *b a' f o''*. *b c''. q''*. (having firſt drawn the ſines *f o*, and *d e*, and

comparing this with the former (at the first beginning of this subject) for the point *h*, which



there fell on that side of the vertical meridian *z x* towards *q*, here falls on the other side towards *s*, likewise the hour of 12, that is *b c*, did there fall betwixt the axis and the substile, but falls here betwixt the substile and the horizontal meridian *b a*: yet this notwithstanding the construction is the very same in both.

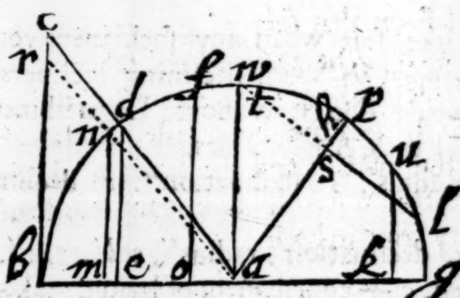
NOTE. 2.

It may be further Noted, that as the Reclination may increase, the points *n*, *c*, *r*, all approach still neerer to *a*, and when the reclination is 90 they

they are all coincident, and this vanisheth into an horizontal dial whose substile will be *b a.*

Also if the Declination be still increased, at last the points *b*, and *m*, will be coincident, and the dial plane will be parallel to the prime vertical, and the work a South dial whose substile is or may be *z a.*

But whether the declination increase or not, if the reclination amount to 70 d. or above, then the substilar line applyed (as before) from *b*, to the vertical *x z*, shall fall between the points *a* and *x*, upon the base of the plane. By which means the former method is rendered insufficient, or (at least) inconvenient in this case. Wherefore having described any semicircle on paper, and made therein the angle *d a b*, equal to the declination



(which let be 50 d.) and the angle *f a b* equal to the reclination (which let be 75) make *b a' f o''.* *b e'' . q''.* (having first drawn the sines *f o*, and *d e*, and

and the tangents *b c*,) so *q* becomes a line known let be *b r* = *q*, and draw *r a* cutting the circle in *n*, draw also *n m*, for the sine of *n b*, and againe make. *b a*′. *m a*″. *o a*′. *x*″. *x* therefore being a line known, may be a sine, as let it be the sine of the arch *g l*, and let *g p* be the latitude 51. 32′. and *l s* the sine of *l p*: make again, *m a*′. *l s*″. *e a*′. *z*″. make *t h* = *z*, the sine of the arch *p t* and make *h a*′. *t a*″. *s a*′. *y*″. Draw radius *a w* perpendicular to *b g*, Lastly make *k u* = *y*, the sine of the arch *g u*: then shall the arch *w n* be the distance of the meridian of the place from the horizon; and the arch *p t* the height of the stile, and the arch *g u* the substilar distance, which are all that were sought.

This, being according to the common way trodden in Trigonometry, I shall not need to prove.

In like sort when any such inconveniency shall happen in South declining incliners, they that would doe it without Logarithmes may work by these Analogismes.

1 Radius′. *s* inclination″. *t* declination′. t. *b*″.

2 *s*. declination′. radius″. s. *b*′. s. *c*″.

Then, *c* + complement of latitude taken from 180 d. let the rest be called *d*.

3 s. *c*′. s. inclination″. s. *d*′. s. *f*″.

4 *t*. inclination′. s. *b*′. t. *f*′. s. *g*″.

Then *b*, is an arch whose complement to 90, is

is the distance betwixt the meridian and horizon.

Also *c*, is an arch which being added to the complement of latitude, and the aggregate taken from a Semicircle, the residue, namely *d*, is an arch composed to find *f*, which is an arch equal to the height of the stile, or Pole above the Plane.

Lastly, *g* or the complement thereof to 90 deg. is an arch equal to the distance of the substile from the meridian of the place.

And these are enough for any man that is but indifferently skilled, to finish the dial with, which being deduced from M. *Wells*, in his Chap. 20. I shall not need to prove. This is for such planes as incline more then the distance betwixt the Zenith and the Æquator.

Almicanters

Are lesser Circles of the Sphære, and may be called the Parallels of declination from the horizon; having in all respects the same relation and habitude to the Azimuths, that the Signes have to the meridians, although these are accompted by 15 d. and those usually by 10 d. and therefore as in the description of the Signes an horizontal dial proper to the plane being first (obscurely) delineated, it was shewed that the points through which the signes or Parallels must passe upon every hour, might be had by applying the tangents of the complements of the Suns height at those hours in those Paralles, from the foot of the per-

pendicular

and the tangents $b\,c$,) so q becomes a line known let be $b\,r = q$, and draw $r\,a$ cutting the circle in n, draw also $n\,m$, for the sine of $n\,b$; and againe make. $b\,a'.\ m\,a''.\ o\,a'.\ x''.$ x therefore being a line known, may be a sine, as let it be the sine of the arch $g\,l$, and let $g\,p$ be the latitude 51: 32'. and $l\,s$ the sine of $l\,p$: make again, $m\,a'.\ l\,s''.\ e\,a'.\ z''.$ make $t\,h = z$, the sine of the arch $p\,t$ and make $h\,a'.\ t\,a''.\ s\,a'.\ y''.$ Draw radius $a\,w$ perpendicular to $b\,g$, Lastly make $k\,u = y$, the sine of the arch $g\,u$: then shall the arch $w\,n$ be the distance of the meridian of the place from the horizon; and the arch $p\,t$ the height of the stile, and the arch $g\,u$ the substilar distance, which are all that were sought.

This, being according to the common way trodden in Trigonometry, I shall not need to prove.

In like sort when any such inconveniency shall happen in South declining incliners, they that would doe it without Logarithmes may work by these Analogismes.

1 Radius'. s inclination''. t declination'. t. b''.

2 s. declination'. radius''. s. b'. s. c''.

Then, c + complement of latitude taken from 180 d. let the rest be called d.

3 s. c'. s. inclination''. s. d'. s. f''.

4 t. inclination'. s. b''. t. f'. s. g''.

Then b, is an arch whose complement to 90, is

is the diſtance betwixt the meridian and horizon.

Alſo *c*, is an arch which being added to the complement of latitude, and the aggregate taken from a Semicircle, the reſidue, namely *d*, is an arch compoſed to find *f*, which is an arch equal to the height of the ſtile, or Pole above the Plane.

Laſtly, *g* or the complement thereof to 90 deg. is an arch equal to the diſtance of the ſubſtile from the meridian of the place.

And theſe are enough for any man that is but indifferently skilled, to finiſh the dial with, which being deduced from M. *Wells*, in his Chap. 20. I ſhall not need to prove, This is for ſuch planes as incline more then the diſtance betwixt the Zenith and the Æquator.

Almicanters

Are leſſer Circles of the Sphære, and may be called the Parallels of declination from the horizon; having in all reſpects the ſame relation and habitude to the Azimuths, that the Signes have to the meridians, although theſe are accompted by 15 d. and thoſe uſually by 10 d. and therefore as in the deſcription of the Signes an horizontal dial proper to the plane being firſt (obſcurely) delineated, it was ſhewed that the points through which the ſignes or Parallels muſt paſſe upon every hour, might be had by applying the tangents of the complements of the Suns height at thoſe hours in thoſe Paralles, from the foot of the perpendicular

pendicular stile to the respective hours; so here making use of that Azimuth which is perpendicular to the Plane; (which in all planes is that which passeth through the foot of the perpendicular stile) the rest of the Azimuths being also inscribed, the tangents complements of the Suns height above the plane, when he is in any Azimuth applyed from the foot of the stile to the said Azimuth gives a point, through which that height, or almicanter upon that Azimuth must passe.

Example, in the Triangle *p z s*, let there be given the complement of Elevation *p z*, the complement of the Declination *p s*, and the Azimuth *p z s*, to find the complement of the Sun's height *z s*.

Suppose the side *s z* continued till a Meridian from *p*, cut it at right angles in *c*.

Then first it is t. c. *p z*′. Radius″. s. c. *p z c*′. t. *z c*″.

Secondly, s. c. *p z*′. s. c. *p s*″. s. c. *z c*′. s. c. *s c*″.

So *z e* and *s e*, being ſeverally found, the difference betwixt them namely *s e* — *z e*, is the complement of the Suns height above the Horizon.

Then find how high the Sun is above the plane of the dial at the ſame time, the tangent complement of that height applyed, from the ſtiles foot to the Azimuth repreſenting the angle *p z s*, gives upon it the Almicanters point, or paſſage.

Or becauſe *s, p s'. s, p z s''. s, z s'. s, z p s''.* the hour from Noon, that is the angle *z p s*, is found; which will croſſe the azimuth aforeſaid in the ſame point alſo.

Which hour if it be uneven, and unfit to remain with the reſt, may be drawn obſcurely.

Of the Jewiſh, Babyloniſh and Italian hours.

The Babyloniſh are accompted equal hours from Sun riſing, and may be inſcribed upon any Plane by help of thoſe two parallels, which ſhew the longeſt, and ſhorteſt day conſiſting of intire hours, as here 16 and 8 hours, and of the Æquator; for a line drawn through the hour of 5 in the firſt, 7 in the æquator, and 9 in the other, is the hour of 1 from Sun riſing.

Likewiſe in the ſame order, through 6. 8, and 10 ſhall paſſe the hour of 2, the like order in all.

In Winter when the parallel of 8 hours ſhall faile, the other two points will ſerve; becauſe the hours to be drawn are right lines.

But after the firſt ſix hours are inſcribed, the Æ-

 qua-

quator failing also, some other diurnal arch as of 9 or 10 hours must be described to supply that want.

The Italian hours are accompted 1, 2, 3, &c. from Sun setting; to inscribe these the same diurnall arches will serve, and a line drawn through them in the hours 9. 7. and 5. after noon, (observing the same order as before) shall be the hour of 1. the like through 7. 5. and 3. shall be the hour 23, the night hours of 9, 10, &c. are the morning hours produced.

The Jewish hours are reckoned like the Babylonish, from Sun-rising, but unequally, their sixt hour being noon; and every hour a twelfth part of the artificial day, of what length soever that be.

The vulgar hours proper to the Plane being first drawn, and the Diurnal arches of 15. 12. and 9. (if it may be) divide the degrees in each by 12. and the quotients by 15. or else (which is all one) divide the said arches by 180, the three quotients shall give the just times in hours or usual parts of hours from 12 of clock upon the two Parallels and the æquator; through which lines drawn by a Ruler shall be the Jewish hours desired.

Example, in latitude 51. 32′, the diurnal arch of 15 hours, is in degrees 225, which divided by 180 the quotient is 1 $\frac{1}{4}$. and so much the Jewish hours of 5 and 7 are distant from noon, an hour and quarter being a twelfth part of the diurnal arch of 15 hours, which hour and quarter being doubled, gives the place of 4 and 8, tripled the

place

place of 3 and 9, &c. from noon, upon that parallel of 15 hours.

In like manner the diurnal arch of 9 hours, is 135 d. which divided by 180, quotient is $\frac{135}{180}$ that is $\frac{3}{4}$ of an hour, which shews the place of the Jewish 7 and 5, to be three quarters after or before noon, and doubled is 1 $\frac{1}{2}$. which gives the place of 8 and 4, all one with our 1 $\frac{1}{2}$. and 10 $\frac{1}{2}$. and so tripling and quadrupling and quintupling of $\frac{3}{4}$ gives the places of the other hours on this parallel of 9 degrees.

And these parts doubled and tripled as is said, will always (in this parallel and the former) fall upon even hours halves or quarters of our hours, which is the onely reason why these two parallels of 15 and 9 are preferred; there being no necessity of using them more then the tropiques or other parallels, onely this conveniency of even parts.

Lastly, in the diurnal arch of 12. that is, the Æquator the equall and unequal hours concurre, that is, the Jewish hours of 5 and 7, with our hours of 11 and 1, so their 4 and 8, with our 2 and 10, &c. so that a line drawn from 1 $\frac{1}{4}$. in the arch of 15 to 1, in the æquator, and from thence to $\frac{3}{4}$ in the arch of 9 is their 7, &c.

The *Circles of Position* I omit, not for that the inscription of them in any plane is difficult, but where the labour is *not much*, and the use of the thing *not any*, I hold that labour *too much*.

The way to describe dials upon Rings, Qua-

drants and Cilinders, as alſo Globes, and Concave-Hemiſpheres, I alſo paſſe over: not for the ſame reaſon as the other; for of all theſe there is much uſe, and pleaſure in uſing; but becauſe every man that ſhall have travailed through dialling, on planes, with the dreſſes thereto belonging, cannot poſſibly want ſo much ingenioſity, as may direct him to doe theſe without book.

CHAP. XVIII.

IT remaines onely to ſay ſomthing of the form of the Parallels upon Planes; which (when they are not circles, (as under the Poles) are all and always Conique Sections, as is ſhewed by *Mydorgius lib.* 4. *Prop.* 34. And here ſhall be ſhewed how at any time or place it may be known what Section it is; although this is not neceſſary, for the Dialler to know; becauſe without knowing them he may draw them upon the plane, as hath been ſhewed already.

Take therefore theſe three briefe Rules, which by *Aguilonius lib.*6. *Prop.* 83. are proved.

Rule. 1.

When the Sun is in any Parallel, if the plane of the dial be parallel, to a great Circle of the Sphere which toucheth the parallel, and the oppoſite thereto, the projection of the ſhadow is a Parabola.

Rule

Rule. 2.

If the Dial plane be equidiſtant, to a great Circle which cuts the parallel, and the oppoſite, the ſhadow runs in an Hyperbola.

Rule 3

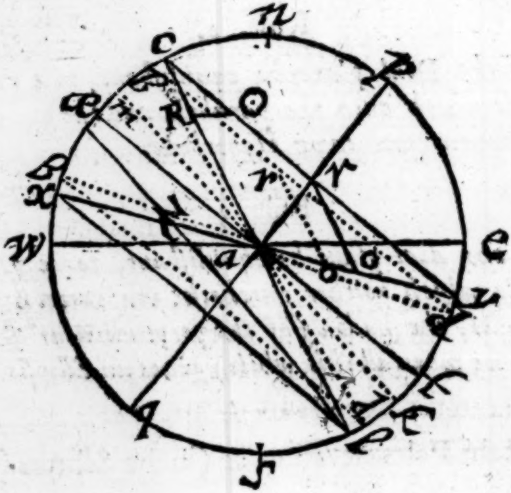
If the dial plane be equidiſtant, to a great Circle which neither toucheth, nor cutteth the Parallel, the ſhadow (of the perpendicular ſtile, for ſo we mean all this while) gives an Ellipſis.

Example.

Let the Poles elevation be (in the Scheme follow,) the arch *e p.*

The Horizon *w e.*
The Meridian *w n e s.*
The Æquator *a t.*
c l the Ecliptique.
c y, *x l* the Tropiques.
b d any other Parallel.
n, *s*, Zenith and Nadir.

Now therefore it is clear, that the horizontal dial in latitude 51. 32. being equidiſtant to the great circle *w e*, which cuts the tropiques, and all the parallels between them, as *b d*, or any other, is (according to that which hath been ſaid) ſuch, as that the ſhadow thereupon all the year long ſhall deſcribe Hyperbolaes; but of different kinds, as it ſhall cut ſeveral parallels more or leſſe unequally.



But if a dial were made parallel to the Eclip-tique *e l*, which toucheth the Tropiques, the sha-dows thereon when the Sun is in the Tropiques would be Parabola's.

Lastly, if the plane were equidistant, not to the æquator *a t*, but to some other plane *m k*, whose great circle neither touches any of the parallels, nor cuts them, the shadow there shall always trace some Ellipsis; not always the same, but lesser, as the Sun, or his parallel approacheth towards the æquator: but greater in those horizons which make more acute angles with the æquator; until at last the horizon and the æquator being coinci-dent,

dent, the projecture of the ſhadow ſhall be a Circle.

Likewiſe the horizon howſoever ſituate, if the Sun be in no Parallel, the Projecture is a right line.

It ſhall not need to bring hither the demonſtrations, which *Aguilonius* uſeth to prove all this; for the whole matter with ſmal a doe may be manifeſted thus.

The Sun being in the ſouthern ſignes, ſuppoſe the darke Cone *c a y*, in North latitude to be cut by a plane *c a y*, through the Vertex *a*, perpendicular to the center of the baſe, it gives the triangle *c a y*, for the flat and under ſuperficies of the ſemicone *c a y*. And let *r o*, be the horizon, or dial plane, (for every dial plane is parallel to ſome horizon) and let it be equidiſtant to *c a*; then the commune ſection *r o*, (which while the Sun is in the tropique *x*, diſtinguiſheth, or rather ſeparateth the light from the darkneſſe) is by the Definition general, of Chap. 11. A Semiparabola, in like ſort might be proved, if the Sun were in any other parallel, as ſuppoſe *b d*, for ſuppoſing the pricked line *r o*, to repreſent the dial plane, parallel to *b a*, the ſame concluſion follows by the ſaid Definition.

Secondly, let the dial plane be *R* ⊙, parallel to the great circle *w e*, which cuts the parallel *c y*, in ⊙, then by the ſaid Definition general, the commune ſection *R* ⊙, which that day ſeparates

the light and darkneſſe upon the Plane, is a ſemi-hyperbola: or if in ſtead of the ſemicone, one conceive the ſection of the Cone through the vertex and axis, to be the plane triangle *c a y*, and *R* ☉ as a right line, to be onely the diameter of the ſection, the thing is the ſame, and the ſection by the ſecond of the third definitions of *Mydorgius*, an Hyperbola.

Laſtly, (to avoid confuſion of lines) let the Sun be in the northern ſignes, and *x a l* the darke Cone, and *z l* the dial plane in South latitude equidiſtant to the great circle *m k*, which neither toucheth nor cutteth the parallels, it is evident that *z l*, or any line equidiſtant to *m k*, ſhall cut the triangle *x a l*, made by a ſection from the vertex, as before, in both the ſides *x a*, and *l a*, and is therefore by the third of the third Definitions of *Mygorgius* the diameter of an Ellipſis, or conceiving all by the general definition of Chap. 11. as before, it is half an Ellipſis.

And ſo any other line parallel to *z l*, and greater then *x l*, is the diameter of a greater Ellipſis, or rather an Ellipſis of a greater Cone, which might be made by producing the lines *a x*, and *a l* at pleaſure.

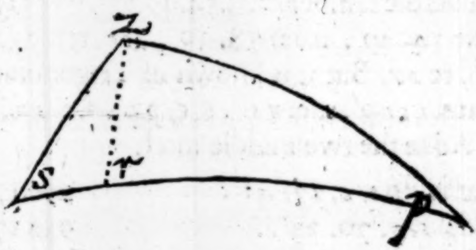
And this is proofe enough, for all that hath been ſaid of this matter; and enough hath been ſaid (I hope) to make it intelligible.

I will now ſhew how the Suns height for every hour of the day, in the beginning of any ſigne, within any latitude may be found. Let

Let the Latitude of the Place, or the dial plane, be 19. 25′. and the ſigne the beginning of *Taurus*, declining 11. 30. as in Chap. 17.

And let it be required to find the Suns height, at any hour that day in that place.

Suppoſe any ſpherical triangle fit for the purpoſe, (no matter for exact delineation, ſo there be ſomthing to help the fancy,) as here the triangle *z p s*, wherein let *p* be the pole of the Æquator, *z* the Zenith of the place, and *s* the Sun; and



z p s the diſtance betwixt the meridian of the Sun *s p*, and the meridian of the place *z p*, that is, the hour from noon.

The eaſieſt way to work, is by Logarithmes of the ſines and tangents, becauſe ſo Addition and Subtraction ſupplies for Multiplication and Diviſion.

The ſide *p z*, is the complement of the latitude, and is 70. 35′. likewiſe the ſide *p s*, is the complement of the declination, and is 78. 30′. and the ſide *s z*, is the complement of the Suns height,

or

or the complement of the thing required. Now then suppose the perpendicular *zr*, to cut *p s* in *r* at right angles; also suppose the hour required to be 4 or 8 of clock, and 4 an afternoon hour, then *z p s* = 60. which is given, to find *s z*, or rather the complement of *s z*.

In the right angled triangle *z r p*, choose the angle *z p r*, for the middle part. Then it will be

s. c. 60 + radius, whose log. are	196989700
Equal t. c. *p z* + t. *p r* subst. t. c. *p z* log.	95471377
Remaines the tan. of *pr* = t. 54. 49. log.	101518323

Take 54. 49′. from 78. 30′. the rest 23. 41′. is equal to *s r*. But it is known in Trigonometrie.

That s. c. *r p*′. s. c. *r s*″. s. c. *p z*′. s. c. *s z*″.

Adde the two middle most.

That is, to *s. c.* 23. 41′.	99617909
Adde *s. c.* 70. 35′.	95217074
Summe is =	194834983
From which take s. c. *r p* (54. 49′.)	97605692
Remaines s. c. *s z* = 31. 53.	97229291

Which is the height of the Sun above that Horizon, at 4 after noon or 8 before noon. And so at two operations, may any altitude, for any hour given, above this, or any other horizon be certainly found. If any like the natural sines and tangents better, he hath three things given to find a fourth, for first.

t. c.

t. c, pz'. s. c. rpz''. Radius' t. pr''. Secondly s. c. pr'. s. c. sr''. s. c. pz'. s c. sz''. as before.

The perpendicular zr, ſhall ever (for the hour betwixt 6 and noon) fall within the triangles, becauſe the angles zsp, and zps are both acute, as in Note 2. of Definition 2.

At 6 of clock the angle zps, is 90 d. and s. c. zs + r = s. c. pz + s. c. ps.

I put r, for Radius, always for abridgment.

At the hours after 6 at evening, or before in morning the ſaid angle zps, is obtuſe; then the ſame triangle remaining.

It is, s. c. pr'. s. c. pr + ps''. s. c. zp'. s. c. zs''.

The arch pr, being firſt found, as before. Or it is the ſame in hours equidiſtant from 6.

A.

A Table of semidiurnal arches, in the beginning of every signe for 32 diverse Latitudes.

Po. alt			♋		♊ ♌		♈ ♎	♓ ♍		♒ ♐		♑	
D.	d.	′	d.	′	d.	′	d.	d.	′	d.	′	d.	′
35	107	44	104	56	98	11	90	81	49	75	4	72	16
36	108	25	105	30	98	30	90	81	30	74	30	71	35
37	109	8	106	6	98	49	90	81	11	73	54	70	52
38	109	52	106	42	99	9	90	80	51	73	18	70	8
39	110	37	107	20	99	29	90	80	31	72	40	69	23
40	111	24	107	59	99	50	90	80	10	72	1	68	38
41	112	12	108	39	100	11	90	79	49	71	21	67	48
42	113	3	109	21	100	33	90	79	27	70	39	66	57
43	113	55	110	4	100	56	90	79	4	69	56	66	5
44	114	50	110	49	101	24	90	78	40	69	11	65	10
45	115	46	111	35	101	44	90	78	16	68	25	64	14
46	116	46	112	24	102	10	90	77	50	67	36	63	14
47	117	48	113	14	102	36	90	77	24	66	46	62	12
48	118	53	114	7	103	4	90	76	56	65	53	61	7
49	120	1	115	2	103	32	90	76	28	64	58	59	59
50	121	13	116	0	104	2	90	75	58	64	0	58	47
51	122	29	117	1	104	33	90	75	27	62	59	57	31
52	123	49	118	6	105	6	90	74	54	61	54	56	11
53	125	15	119	14	105	40	90	74	20	60	46	54	45
54	126	46	120	26	106	16	90	73	44	59	34	53	14
55	128	23	121	42	106	53	90	73	7	58	18	51	37
56	130	8	123	3	107	33	90	72	27	56	57	49	52
57	132	2	124	31	108	15	90	71	45	55	29	47	58
58	134	6	126	4	109	0	90	70	00	53	56	45	54
59	136	21	127	46	109	47	90	70	13	52	14	43	39
60	138	52	129	35	110	38	90	69	22	50	25	41	8
61	141	40	131	39	111	32	90	68	18	48	25	38	20
62	144	52	133	47	112	30	90	67	30	46	13	35	8
63	148	35	136	13	113	32	90	66	28	43	47	31	25
64	153	3	138	58	114	39	90	65	21	41	2	26	57
65	158	49	142	6	115	52	90	64	8	37	54	21	11
66	167	35	145	44	117	11	90	62	49	34	16	12	25

A Table of the amplitude in the beginning of every ſigne, for 27. Elevations of the Pole.

Eleva. of the Pole D.	♋ d.	♋ ′	♊ ♌ d.	♊ ♌ ′	♉ ♍ d.	♉ ♍ ′
36	29	29	25	13	14	15
37	29	55	25	34	14	26
38	30	21	25	57	14	38
39	30	49	26	20	14	51
40	31	19	26	48	15	4
41	31	51	27	11	15	18
42	32	24	27	38	15	32
43	32	59	28	7	15	48
44	33	37	28	38	16	4
45	34	16	29	11	16	21
46	34	59	29	45	16	39
47	35	43	30	22	16	58
48	36	31	31	1	17	19
49	37	22	31	42	17	40
50	38	17	32	26	18	3
51	39	15	33	13	18	27
52	40	18	34	3	18	52
53	41	26	34	57	19	19
54	42	39	35	55	19	48
55	43	58	36	57	20	19
56	45	24	38	4	20	51
57	46	59	39	16	21	26
58	48	43	40	35	22	4
59	50	38	42	1	22	44
60	52	47	43	35	23	28
61	55	13	45	20	24	15
62	58	1	47	15	25	5

♈ ♎

In theſe 2 ſignes there is no ampl.

In the other oppoſite ſignes ♏, ♓; ♐, ♒ and ♑, the ſame Table may ſerve, giving to oppoſites equal amplitude as to ♑, in each degree the ſame as to ♋, to ♐ and ♒, the ſame as to ♊ and ♌, laſtly to ♏ and ♓, the ſame as to ♍, and ♉.

A Table of arches of the Horizon intercepted betwixt the Meridian, and each hour line upon the Diall Plane, for Horizontall and Verticall Dials, Calculated for 21 Elevations of the Pole.

The Altitude of the Pole for Horizontals.	12		11.1.		10.2.		9. 3.		8. 4.		7. 5.		6.	The Altitude of the Pole in Verticals.
	d.	′	d.	′	d.	′	d.	′	d.	′	d.	′	d.	
35			8	43	18	18	29	49	44	49	64	35	90	55
36			8	57	18	46	30	32	45	30	65	29		54
37			9	10	19	9	31	2	46	11	66	0		53
38			9	22	19	34	31	37	46	50	66	29		52
39			9	33	19	58	32	11	47	28	66	55		51
40			9	45	20	21	32	44	48	7	67	21		50
41			9	57	20	44	33	16	48	39	67	47		49
42			10	10	21	7	33	46	49	12	68	11		48
43			10	22	21	29	34	18	49	44	68	33		47
44			10	32	21	51	34	47	50	16	68	54		46
45			10	43	22	12	35	17	50	46	69	15		45
46			10	54	22	33	35	44	51	15	69	35		44
47			11	5	22	53	36	11	51	42	69	53		43
48			11	17	23	13	36	37	52	9	70	11		42
49			11	25	23	33	37	3	52	35	70	28		41
50			11	35	23	52	37	28	53	0	70	43		40
51			11	45	24	9	37	52	53	24	70	59		39
52			11	55	24	27	38	15	53	46	71	13		38
53			12	5	24	43	38	37	54	8	71	28		37
54			12	13	25	2	38	58	54	29	71	41		36
55			12	22	25	18	39	19	54	49	71	54		35

Theſe Tables need no explanation, the uſe of them being evident. But if they prove not ſatisfactorie, for want of calculation, for further degrees of Elevation; or for want of halves, and quarters of degrees, or the like of hours, they are as I had them out of *Kercherus* his *Ars Magna.* Neverthelesſe I will ſhew the making of them, whereby any man may fit them for his own purpoſe, and for his place, (if it happen without theſe limits) by his own calculation, as followeth.

First for the Table A.

Radius$'$ *t. c.* Elevation$''$. *t. c*$'$. Declination$'$. *b*$''$.

This *b*, is the ſine complement of the angle at the Pole, which ſhews the hour from Noon, in Winter; and the hour from Midnight in Summer, wherein the Sun riſeth, having declination, which declination is ready in tables, the making of which ſhall be ſhewed anon and alſo the table of the Suns declination at the end hereof ſhall enſue.

The angle at the Pole, ſo found being divided by 15. ſhewes in Winter the ſemidiurnal arch in hours, which was had by the firſt working in degrees and minutes.

And in Summer the ſeminocturnal; whoſe complement to 180 degrees or to 12 hours, is the thing required, here all the time from the vernal to the Autumnal Æquinoctial, is called Summer.

Secondly

Secondly, for the Table B.

s. c. Elevation′, *s.* Declination″, Radius′: *s. c.* Azimuth″, which Azimuth being compared with 90 d. difference is the Amplitude.

Example, for Elevation 40, *initio* ♊

To Radius log.	10000000
Adde Sine declination 20. 13′. log.	09538537
Summe	19538537
Subtract sine complement elevation,	9884254

Remaines
Sine complement Azimuth, which complementhere is the Amplitude } 9654283

The arch belonging to sine 9654283. being sought in the Canon, is 26. 48′. which is the Amplitude required, where the Elevation is 40 degrees.

Thirdly, for the Table C.

First, to find any horizontal arch for any hour; as for Example 3, or 9.

Radius′. *t.* of the hour in degrees; that is here *t.* 45″.

Sine Elevation′. *t.* of the arch required″.

Or else, *t. c.* 45′. *r*″. *s.* Elevation′. *t.* arch required″.

Secondly, in a Prime Vertical.

Radius′. *t.* hour″. *s. c.* Elevation′. *t.* of the arch″.

It is at first in the Symbols, Chap. 1. advertised

that

that the letters *s.* and *t*; or *s. c.* and *t c.* signifie the sine, and tangent; or sine complement and tangent complement of an arch or angle.

And working by the Logarithmes of the sines and tangents, the former Analogismes happen not but in their stead certain Æqualities, or Æquations, as follow.

For the Table A.

t c. Elevation + *t c.* declination = Rad: + *b.*

Which *b*, is the co-sine of the thing required, that is, of the angle at the Pole, which divided by 15, gives the time.

For the Table B.

Rad: + *s.* declination = *s c.* elevation + *s c.* Azimuth.

For the Table C.

s. Elevation + *t.* the hour = Radius + *t.* the arch, or more readily thus, *t c.* hour′ *r*″ *s.* Elevat′ *t.* of arch from the substile″.

It must still be remembred that *r*, stands for Radius.

The Elevation is always taken for the height of the Pole above the horizon, which horizon is the dial plane.

In other planes, as the Prime Vertical, and all other verticals, the height of the Pole above the plane must be used, having therefore found that, call it *p*, or else call the declination *q*, then *s.q* + *s.* elevat. = *r* + *s. p.*

Q And

And, $r + s, p, = t\,c.$ the hour $+ t,$ the arch of that hours diſtance from the ſubſtile.

So after ſtill till all the hours be found, this later work muſt be repeated.

Whereas we uſe $t\,c.$ the hour, and $t.$ the arch, by the hour is always meant the angle at the Pole, or the ſpace there included between any hour, and the ſubſtile; as 15 d. for 1, 30 d. for 2, &c. The arch is the diſtance of any hour from the ſubſtile meaſured in the arch of a circle, whoſe center is the center of the dial, when it is projected upon the plane.

To find the Declination of a Place.

The declination is an arch of a great circle paſſing through the Poles of the World, and the center of the place whoſe declination is ſought, intercepted between the ſaid center and the Æquator.

If the place have no latitude, that is, if it be in the Ecliptique, the neereſt diſtance from *Aries* or *Libra* being given, call it $b.$

Then r' s. 23 d. 32 m$''$ s. b' s. e' and $e,$ is the declination required: working by the Natural Sines.

Secondly, if the place have latitude, that being given, or found in Tables, and the right angle which the circle of latitude makes with the Ecliptique, (for all circles of latitude do ſo, as the circles of declination do with the Æquator) and the next

next diſtance to *Aries* or *Libra* being alſo given.

1 Then if the place lie betwixt the Ecliptique and the æquator, call the neereſt diſtance to ♈ or ♎, *b*, as before.

And the latitude given *c*. It is Logarichmically *s*. $c + r = s, b + s, a$ & *a*, is an angle, which being taken out of 23.32′. leaves an angle, which angle call *d*, then $s, b + s, d = r + s, e$.

And *e*, is the declination required.

2 If the place lie betwixt the Ecliptique and the Pole, the angle *a*, found as before, muſt be added to 23.32′. and call the ſum *f*, then $s, b + s, f = r + s, e$, &c.

3 Laſtly, let the place lie betwixt the æquator and the other Pole, then $s, c + r = s, b + s, a$, and from *a*, ſubtracting 23.32′. call the reſt *g*.

Then $s, b + s, g = r + s, e$, and *e*, the declination.

To find the right Aſcenſion of a Place.

If it be in the Ecliptique, as the Sun is, let the neereſt diſtance from *Aries* be called ſtill *b*. And working by Logarithms, it is $r + s\,c, 23.32' = t\,c, b + t, a$.

And *a* is the right Aſcenſion.

2 If the place have latitude, call it ſtill *c*, and let the declination found with latitude, as before, be called, *q*.

Then $s\,c, b, + r = s\,c, q + s\,c, a$ and *a*, is the right aſcenſion; or (between ♋ and ♎) the complement of it.

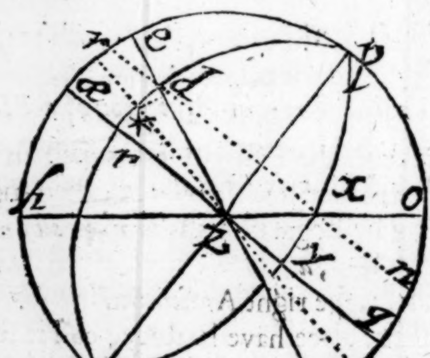
To find the Ascensional Difference.

Thus t. elevation + t. declination = r + $s.y$. And y, is the Ascensional difference.

To find the, Oblique Ascension.

In the southern signes adde the Ascensional difference to the right Ascension: or in the northern signes subtract the same from the right Ascension, the sum in Winter, and the remainer in Summer, is the oblique Ascension.

That which hath been last said concerning the Declination, Right Ascension, Ascensional diffeence and oblique Ascension, may be illustrated, and demonstrated also out of this figure, at least with some small variationr.



In which let *h o*, be the horizon, *a q*, the æquator,

p p the Axis of the World, *e c* the Ecliptique, *m n*, the parallel in which the Sun is at *d*.

p o the Elevation, *z* the points ♈ and ♎, *b z*. the neerest distance from one of those points.

Which is supposed given, or known, *d r* the declination.

z r the right Ascension of the Sun: or sometimes the complement of it.

The angle *x p o*, is the complement of the Ascensional difference, *y q* the measure of it.

Therefore here in Winter *r y*, is the oblique ascension, but the Sun being now supposed to be in *d*, that is 20 degrees of *Taurus*, by that which hath been said before, *r z* — *z y*, is the oblique ascension.

All these may be found (enough being given) as first in the triangle *d r z* right angled at *r*, are given the side *d z* = 5 cd. the angle *d z r* = 23.32′. and the angle *r* 90 d. to find the declination *d r*. Working by the Artificial sines and tangents, put *b r*, for the middle part, therefore, *s. d r* + radius, = *s. d z* + *s. d z r*, which resolved into proportionalty by the 14. of the 6. of *Euclide*, will be.

Radius′ *s. d z*″ *s. d z r*′ *s. d r*″ or,

Alternately, Radius′ *s. d z r*″ *s. d z*′ *s. d r*″, the thing sought. And the Analogisme the very same with that shewed before for finding the declination.

Then for the right ascension, *z r* make *d z r* the middle part, it is *s c. d z r* + Radius = *t c. d z* + *t. z r*.

And

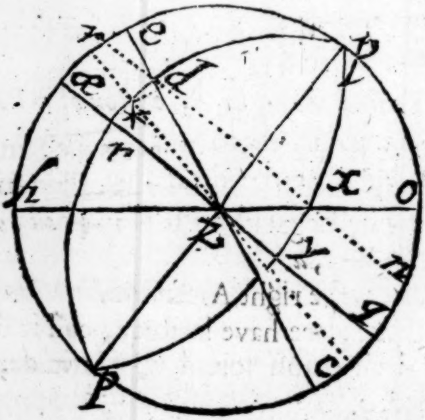
To find the Aſcenſional Difference.

Thus t. elevation $+$ t. declination $= r + s.y$. And y, is the Aſcenſional difference.

To find the, Oblique Aſcenſion.

In the ſouthern ſignes adde the Aſcenſional difference to the right Aſcenſion: or in the northern ſignes ſubtract the ſame from the right Aſcenſion, the ſum in Winter, and the remainer in Summer, is the oblique Aſcenſion.

That which hath been laſt ſaid concerning the Declination, Right Aſcenſion, Aſcenſional diffeence and oblique Aſcenſion, may be illuſtrated, and demonſtrated alſo out of this figure, at leaſt with ſome ſmall variationr.



In which let h o, be the horizon, a q, the æquator,

p p the Axis of the World, *e c* the Ecliptique, *m n*, the parallel in which the Sun is at *d*.

p o the Elevation, *z* the points ♈ and ♎, *b z*. the neerest distance from one of those points.

Which is supposed given, or known, *d r* the declination.

z r the right Ascension of the Sun: or sometimes the complement of it.

The angle *x p o*, is the complement of the Ascensional difference, *y q* the measure of it.

Therefore here in Winter *r y*, is the oblique ascension, but the Sun being now supposed to be in *d*, that is 20 degrees of *Taurus*, by that which hath been said before, *r z* — *z y*, is the oblique ascension.

All these may be found (enough being given) as first in the triangle *d r z* right angled at *r*, are given the side *d z* = 5 0 d. the angle *d z r* = 23.32′. and the angle *r* 90 d. to find the declination *d r*. Working by the Artificial sines and tangents, put *b r*, for the middle part, therefore, *s. d r* + radius, = *s. d z* + *s. d z r*, which resolved into proportionalty by the 14. of the 6. of *Euclide*, will be.

Radius′ *s. d z*″ *s. d z r*′ *s. d r*″ or,

Alternately, Radius′ *s. d z r*″ *s. d z*′ *s. d r*″, the thing sought. And the Analogisme the very same with that shewed before for finding the declination.

Then for the right ascension, *z r* make *d z r* the middle part, it is *s c. d z r* + Radius = *t c. d z* + *t. z r*.

And

And resolved, *t c. d z.*′Rad″: *s c. d z r*′ *t. z r*″.
That is Radius′ *t. d z*″ *s c. d z r*′ *t. z r*″.
which is the same Analogisme with that before, for finding the right ascension.

Secondly for the ascensional difference in the triangle *x p o*, right angled at *o*, put the angle *xpo*, for the middle part.

Then *s c. x po* + Radius = *t.po* + *tc.x p* but *t c. x p* = *t. z y.*

That is, Rad′ *t.p o*′′ *t.x y.*(or *br*′) *s c. x p o*′′

And the complement of *x p o*, is *x p z*.

Whose measure is *z y*, the ascensional difference sought for.

Example of all, first for the declination *r d*, put *r d* for the middle part.

Unto, *s. d z*, 50.	log.	9884254
Adde, *s. d z r*, 23½.	log.	9601570
Summe is		19485824

And subtracting Radius remaines. 9485824

Which is the sine of 17.49′. for the declination.

Then for the right Ascension *r z*, put that for the middle part, and

Unto, *t. d r*, 17.49.	log.	9507027
Adde, *t c. d z r*, 66.28	log.	10361007
Summe		19868034

Whence subtracting Radius, remain. 9868034

The sine of 47.33′. for *r z*: the right ascension.

Secondly,

Secondly, for the aſcenſional difference *z p x*, in the triangle *x p o*, putting the angle *x p o*, for the middle part,

Unto *t. p o*, 51. 32′. log. 10099913
Adde, *t c. p x*, that is *t. b r*, 17. 49′. log. 09507027

Summe is 19606940
From which taking Radius, reſts 09606940

The Sine complement of *x p o*, or the ſine of *x p z* = 23. 51′. the Aſcenſional difference. Which being taken from *r z*, the right aſcenſion 47 33. remaines *r z* — *z y* = 23.42. for the oblique aſcenſion.

Note 1.

The complement of the aſcenſional difference is equal to the quantity of hours and parts betwixt midnight and ſunriſing.

NOTE. 2.

The oblique aſcenſion of the Sunne being taken, from the right aſcenſion in Summer, the reſidue is equal to the exceſſe, whereby the ſemidiurnal arch is more then 6 hours : alſo the right aſcenſion taken from the oblique aſcenſion in Winter, the reſt is the defect whereby the ſemidiurnal arch is leſſe then 6 hours. And the right and oblique aſcenſion are neerer to æquality, as the Sun attaineth neer either ♈ or ♎.

If the place be not in the Ecliptique, but hath

latitude, as the asterisme * in the last figure, if there be given that latitude d*, and the distance from the next æquinoctial point z*, these with the right angle at d, are sufficient to find out all, in manner as hath been shewed before.

HEre follow two Tables of Interest, whereof the first sheweth how much 100 *li.* with all its increase by meanes of Compound Interest at the severall rates of 5, 6, 7, 8, 9, and 10 *per. Cent.* amounts to annually for 31 years: The second sheweth the like increase for 100 *li.* Annuity or yearly rent, at the like rates, and for the same terme. In both which the first Columne towards the left hand shews the number of years successively to 31, the second gives the increase together with the Principal in intire pounds *sterlin*; the third hath the Numerators of the fractions of a pound to be added to their respective Integers. Onely in the rate for 6 *per. Cent.* (which is now of more frequent use) the fractions are reduced to shillings and pence, (ommitting lesse then pence) as may easily be seen by the table. And the fractions in the other rates (whose common denominator is 1000000) may be easily enough, either so reduced, or very neerly guessed, by such as are but moderately versed in Arithmetique.

ye.	At 5	pe.Cent.	ye.	At 6	p.	Ce.
1	105	000000	1	106		
2	110	250000	2	112	7	2
3	115	762500	3	119	2	0
4	121	550625	4	126	4	11
5	127	628156	5	133	16	5
6	134	009563	6	141	17	0
7	140	710041	7	150	7	3
8	147	745543	8	159	7	8
9	155	132820	9	168	18	11
10	162	889461	10	179	1	8
11	171	033934	11	189	16	7
12	179	585630	12	201	4	4
13	188	564911	13	213	5	10
14	197	993156	14	226	1	9
15	207	892813	15	239	13	1
16	218	287453	16	254	0	8
17	229	201825	17	269	5	6
18	240	661915	18	285	8	8
19	252	695062	19	302	11	2
20	265	329845	20	320	14	2
21	278	596337	21	339	18	9
22	292	526154	22	360	7	0
23	307	152461	23	381	19	5
24	322	510084	24	404	17	10
25	338	635588	25	429	3	11
26	355	567367	26	454	18	11
27	373	345735	27	482	4	10
28	392	013021	28	511	3	7
29	411	613672	29	541	17	0
30	432	194355	30	574	7	2
31	453	804072	31	608	16	5

latitude, as the asterisme * in the last figure, if there be given that latitude d*, and the distance from the next æquinoctial point z*, these with the right angle at d, are sufficient to find out all, in manner as hath been shewed before.

HEre follow two Tables of Interest, whereof the first sheweth how much 100 *li.* with all its increase by meanes of Compound Interest at the several rates of 5, 6, 7, 8, 9, and 10 *per. Cent.* amounts to annually for 31 years: The second sheweth the like increase for 100 *li.* Annuity or yearly rent, at the like rates, and for the same terme. In both which the first Columne towards the left hand shews the number of years successively to 31, the second gives the increase together with the Principal in intire pounds *sterlin*; the third hath the Numerators of the fractions of a pound to be added to their respective Integers. Onely in the rate for 6 *per. Cent.* (which is now of more frequent use) the fractions are reduced to shillings and pence, (ommitting lesse then pence) as may easily be seen by the table. And the fractions in the other rates (whose common denominator is 1000000) may be easily enough, either so reduced, or very neerly guessed, by such as are but moderately versed in Arithmetique.

ye.	At 5	pe.Cent.	ye.	At 6	p.	Ce.
1	105	000000	1	106		
2	110	250000	2	112	7	2
3	115	762500	3	119	2	0
4	121	550625	4	126	4	11
5	127	628156	5	133	16	5
6	134	009563	6	141	17	0
7	140	710041	7	150	7	3
8	147	745543	8	159	7	8
9	155	132820	9	168	18	11
10	162	889461	10	179	1	8
11	171	033934	11	189	16	7
12	179	585630	12	201	4	4
13	188	564911	13	213	5	10
14	197	993156	14	226	1	9
15	207	892813	15	239	13	1
16	218	287453	16	254	0	8
17	229	201825	17	269	5	6
18	240	661915	18	285	8	8
19	252	695062	19	302	11	2
20	265	329845	20	320	14	2
21	278	596337	21	339	18	9
22	292	526154	22	360	7	0
23	307	152461	23	381	19	5
24	322	510084	24	404	17	10
25	338	635588	25	429	3	11
26	355	567367	26	454	18	11
27	373	345735	27	482	4	10
28	392	013021	28	511	3	7
29	411	613672	29	541	17	0
30	432	194355	30	574	7	2
31	453	804072	31	608	16	5

ye.	*At 7*	*pe.Cent.*	*ye.*	*At 8*	*p. Cent.*
1	107	000000	1	108	000000
2	114	490000	2	116	640000
3	122	504300	3	125	971200
4	131	079601	4	136	048894
5	140	255173	5	146	932807
6	150	073035	6	158	687431
7	160	578147	7	171	382355
8	171	818617	8	185	092943
9	183	845920	9	199	900378
10	196	715134	10	215	892408
11	210	485193	11	233	163800
12	225	219156	12	251	816904
13	240	984497	13	271	962256
14	257	853412	14	293	719236
15	275	903151	15	317	216774
16	295	216371	16	342	594116
17	315	881519	17	370	001645
18	337	993225	18	399	601776
19	361	652750	19	431	569818
20	386	968442	20	466	095403
21	414	056231	21	503	383035
22	443	040167	22	543	653677
23	474	052979	23	587	145971
24	507	236687	24	634	117648
25	542	744255	25	684	847059
26	580	736353	26	739	634823
27	621	373898	27	798	805608
28	664	870071	28	862	710056
29	711	410976	29	931	726860
30	761	209744	30	1006	265009
31	814	494426	31	1086	766210

ye.	At 9	p. Cent.	ye.	At 10	p. Cent.
1	109	000000	1	110	000000
2	118	810000	2	121	000000
3	129	502900	3	133	100000
4	141	158161	4	146	410000
5	153	862395	5	161	051000
6	167	710010	6	177	156100
7	182	803911	7	194	871710
8	199	256283	8	214	358881
9	217	189348	9	235	794769
10	236	736389	10	259	374246
11	258	042664	11	285	311671
12	281	266504	12	313	842838
13	306	580489	13	345	227122
14	334	172733	14	379	749834
15	364	248279	15	417	724817
16	397	030624	16	459	497299
17	432	763380	17	505	447029
18	471	712084	18	555	991732
19	514	166173	19	611	590905
20	560	441127	20	672	749995
21	610	880828	21	740	024994
22	665	860103	22	814	027493
23	725	787512	23	895	430242
24	791	108388	24	984	973266
25	862	308143	25	1083	4705 3
26	939	915876	26	1191	817652
27	1024	508305	27	1310	999417
28	1116	714052	28	1442	099359
29	1217	218317	29	1586	309295
30	1326	767966	30	1744	940225
31	1446	177082	31	1919	434247

The ſecond Table for 100 *li.* year rent, or Annuity.

ye.	At 5	p. Cent.	ye.	At 6	p.Cent.	
1	100	000000	1	100		
2	205	000000	2	206		
3	315	250000	3	318	7	2
4	431	012500	4	437	8	7
5	552	675625	5	563	13	6
6	680	309406	6	697	9	11
7	814	324876	7	839	6	11
8	955	041119	8	989	14	2
9	1102	793174	9	1149	1	11
10	1257	932832	10	1318	0	10
11	1420	829473	11	1497	2	6
12	1591	870946	12	1686	19	0
13	1771	464493	13	1888	3	4
14	1960	037717	14	2101	9	2
15	2158	039602	15	2327	10	11
16	2365	941582	16	2567	4	0
17	2484	238661	17	2821	4	7
18	2813	450594	18	3090	10	1
19	3054	123123	19	3375	18	8
20	3306	829279	20	3678	9	10
21	3572	170742	21	3999	4	1
22	3850	779279	22	4339	3	1
23	4142	318242	23	4699	10	0
24	4449	434154	24	5081	9	5
25	4771	905861	25	5486	7	2
26	5110	501154	26	5915	10	10
27	5466	026211	27	6370	9	6
28	5839	327521	28	6852	14	0
29	6231	293897	29	7363	17	3
30	6642	858591	30	7905	14	9
31	7075	001520	31	8480	0	9

The second Table for 100 *li.* year rent, or Annuity.

ye.	At 5	p. Cent.	ye.	At 6	p. Cent.	
1	100	000000	1	100		
2	205	000000	2	206		
3	315	250000	3	318	7	2
4	431	012500	4	437	8	7
5	552	675625	5	563	13	6
6	680	309406	6	697	9	11
7	814	324876	7	839	6	11
8	955	041119	8	989	14	2
9	1102	793174	9	1149	1	11
10	1257	932832	10	1318	0	10
11	1420	829473	11	1497	2	6
12	1591	870946	12	1686	19	0
13	1771	464493	13	1888	3	4
14	1960	037717	14	2101	9	2
15	2158	039602	15	2327	10	11
16	2365	941582	16	2567	4	0
17	2484	238661	17	2821	4	7
18	2813	450594	18	3090	10	1
19	3054	123123	19	3375	18	8
20	3306	829279	20	3678	9	10
21	3572	170742	21	3999	4	1
22	3850	779279	22	4339	3	1
23	4142	318242	23	4699	10	0
24	4449	434154	24	5081	9	5
25	4771	905861	25	5486	7	2
26	5110	501154	26	5915	10	10
27	5466	026211	27	6370	9	6
28	5839	327521	28	6852	14	0
29	6231	293897	29	7363	17	3
30	6642	858591	30	7905	14	9
31	7075	001520	31	8480	0	9

ye.	At 7	p. Cent.	ye.	At 8	p. Cent.
1	100		1	100	
2	207		2	208	
3	321	490000	3	324	640000
4	443	994300	4	449	611200
5	575	073901	5	585	580096
6	715	329074	6	732	426503
7	865	401609	7	891	019623
8	1025	979721	8	1062	301192
9	1197	798301	9	1247	285287
10	1381	644182	10	1446	068109
11	1578	359274	11	1661	753557
12	1788	844423	12	1893	693941
13	2014	063532	13	2145	189456
14	2255	047979	14	2416	804612
15	2512	901337	15	2710	148980
16	2788	804430	16	3026	960898
17	3084	020740	17	3369	117769
18	3399	902191	18	3738	647190
19	3737	895344	19	4137	738965
20	4099	548018	20	4568	758082
21	4486	516379	21	5034	258738
22	4900	572525	22	5536	999437
23	5343	612601	23	6079	959391
24	5817	665483	24	6666	356142
25	6324	902066	25	7299	664633
26	6867	645210	26	7983	637803
27	7448	380374	27	8722	328827
28	8069	767000	28	9520	115133
29	8734	650690	29	10381	724346
30	9446	076238	30	11312	262290
31	10207	301574	31	12317	243213

ye.	At 9	p. Cent.	ye.	At 10	p. Cent.
1	100		1	100	
2	209		2	210	
3	327	810000	3	331	
4	457	312900	4	462	100000
5	598	471016	5	608	310000
6	752	333456	6	769	141000
7	920	043467	7	946	055100
8	1102	847379	8	1140	660600
9	1302	103643	9	1354	726600
10	1509	292970	10	1590	199200
11	1745	129337	11	1849	219100
12	1992	190977	12	2134	141000
13	2271	488165	13	2447	575100
14	2575	922099	14	2792	310610
15	2907	755087	15	3171	541671
16	3209	453045	16	3588	695838
17	3598	303819	17	4047	565421
18	4022	150662	18	4552	321925
19	4484	144223	16	5107	554159
20	4987	717203	20	5718	309575
21	5536	611751	21	6390	140532
22	6134	906809	22	7129	154585
23	6787	048420	23	7942	070043
24	7497	882777	24	8836	277047
25	8272	692226	25	9819	904751
26	9117	234526	26	10901	895226
27	9932	186348	27	12092	084748
28	10932	186348	28	13401	293222
29	12016	083110	29	14841	422544
30	13197	530589	30	16425	564798
31	14485	308342	31	18168	121277

The use of these Tables is thus.

If it be asked how much 100 *li.* put forth to use comes to in 17 years, at 6 *per cent.* compound interest? Look for the title of 6 *per cent.* at the top of the leafe, and in the first Table, then also looke down in the columnn entituled *Years*, till you finde the number 17, just over against 17, towards the right hand you shall finde 269 *li.* 5 *s.* 6 *d.* which is the thing required.

Or if it be asked how much 100 *li. per Annum* in rent amounts to in 13 years at 8 *per cent.* compound Interest? Look in the second Table for 13 years, and under 8 *per cent.* you shall see 2145 *li.* and $\frac{189456}{1000000}$ of a pound, that is reduced, 2145 *li.* 3 *s.* 9 *d.* and something more, which more, being lesse then a peny, I omit, as (in this case) not considerable.

NOTE.

Although I say it amounts to so much, yet I do not say it is worth so much; for who would part with 2145 *li.* 3 *s.* 9 *d.* presently, in hope to get it up again in 13 years by 100 *li. per annum*? When money was at 8 *per cent.* a Lease of 21 years was accounted by some worth 9 $\frac{1}{2}$ years purchase, by others worth 10 years purchase: so that 100 *li. per annum* rent at the most, is worth but 1000 *li.*

in

in 21 years: that is (by the rule of three) *619 li.* 11 *d.* for 13 years.

Note 2.

It is also fit to be known, that proportionally, as Money is lesse valued, land is more, *et contra.* So that according to 10 years, purchase for Rent Charges or Annuities for years, when Money was at 8. *per. Cent.* the money being now at 6. *per. Cent.* the purchase must be 13 years 4 months rent of the land, &c.

So likewise, Money being at 8. *per. Cent.* land for ever used to be sold for 20 years Rent, but now (if no external accident hinder) it ought to be sold for $26\frac{2}{3}$. times the yearly rent thereof.

For $6'\,8''.\,20'\,26\frac{2}{3}$. Also, $6'\,8''.\,10'\,13\frac{1}{3}''$. And the like Analogisme will serve being used in other rates. As if money were at 5 or 7. *per. Cent.* then $5'\,8''.\,20'\,32''$. or $7'\,8''.\,20'\,22\frac{6}{7}''$.

Also $5'\,8''.\,10'\,16''$. Or $7'\,8''.\,10'\,11\frac{6}{14}''$. And so of any.

This, considering the largenesse and clearenesse of the Tables, is all I mean to say concerning that compound interest which is called *Direct* or *Profitable.*

There is another sort of Interst which gives the yearly decrease of 100 *li.* or any other Summe. And this is called Compound Interest *Rebated,* or *Damageable.*

Which *Decrease* is orderly made by subtracting the Interest from the principall yearly, as the *increase* of 100 *li.* in the former Tables was caused by adding the yearly interest.

Example.

If 100 give 6, then at the end of the first year the 100 *li.* is increased, and become 106, and so again, if 100 give 6, what 106? it makes $6\frac{36}{100}$ which adde to 106, then the said 100 *li.* is at the end of two years hereby increased to $112\frac{36}{100}$, and so the first Table is made for every year.

But now if 100 give 6, what shall 94 give? it is $5\frac{84}{100}$, which taken from 94, rests $88\frac{16}{100}$, so the 100 *li.* at the end of the first year is decreased to 94 *li.* and at the end of the second to $88\frac{16}{100}$.

But because the composing Tables for this is much labour as that which hath been done already, also for varieties sake, I will adde a third Table (which I take out of *Simon Stevens Practical Arithmetick*) consisting of artificiall numbers, which wil serve as wel for *direct* as *rebating* interest. And when I have shewed the way to make and use those Tables, and put a few Problemes requisite and difficult, and adjoyned a Table of the Suns Declination, I mean to conclude this Treatise.

Here

Here followeth the third Table.

	The Table for 5. p. C.	for 6. per. Cent.	for 7. ptr. Cent.
1	9523810	9433962	9345794
2	9070295	8899964	8734387
3	8638376	8396192	8162979
4	8227025	7920936	7628952
5	7835262	7472581	7129862
6	7462154	7049605	6663422
7	7106813	6650571	6227497
8	6768393	6274124	5820091
9	6446089	5918985	5439337
10	6139132	5583948	5083493
11	5846792	5267875	4750928
12	5568373	4969893	4440120
13	5303212	4688390	4149645
14	5050678	4423009	3878173
15	4810170	4172650	3624461
16	4581114	3936462	3387347
17	4362966	3713643	3165745
18	4155206	3503437	2958640
19	3957339	3305129	2765084
20	3768894	3118046	2584191
21	3589423	2941553	2415132
22	3418498	2775050	2257133
23	3255712	2617972	2109470
24	3100678	2469785	1971467
25	2953027	2329986	1842493
26	2812407	2198100	1721956
27	2678483	2073679	1609305
28	2550936	1956301	1504023
29	2429463	1845567	1405629
30	2313774	1741101	1313672

	The Table for 8.*p*.*C*.	for 9. *per*. *Cent*.	for 10. *p*. *Cent*.
1	9259259	9174312	9090909
2	8573386	8416800	8264463
3	7938322	7721835	7513148
4	7350298	7084252	6830135
5	6805831	6499314	6209214
6	6301695	5962673	5644740
7	5834903	5470342	5131582
8	5402688	5018662	4665075
9	5002489	4604277	4240977
10	4631934	4224107	3855434
11	4288828	3875328	3504940
12	3971137	3555347	3186309
13	3676979	3261786	2896645
14	3404610	2992464	2633314
15	3152417	2745380	2393922
16	2918905	2518697	2176293
17	2702690	2310731	1978448
18	2502491	2119937	1798589
19	2317121	1944896	1635081
20	2145482	1784308	1486837
21	1986557	1636980	1351306
22	1839405	1501817	1228460
23	1703153	1377814	1116782
24	1576994	1264050	1015256
25	1460180	1159679	0922960
26	1352019	1063926	0839055
27	1251869	0976079	0762777
28	1159138	0895485	0693434
29	1073276	0821546	0630395
30	0993774	0753712	0573086

The construction of these Tables, or any other the like is as followeth.

Having made choise of some great Decimall number, I mean, so it may consist of all Ciphers, except unity towards the left hand, as in these it is 10000000 (which shall be called the Radius of the Tables) this Radius being multiplyed by 100 (which is the principall) and divided by the principall *plus* the Interest, the quotient is the number in the Table for the first year, which quotient being again multiplyed by 100, and the product divided by principall *plus* interest as before, the quotient shall be the number in the Tables for the second year, and so may every years respective number be found, as was the second.

Example in Interest 7 per cent.

First, 10000000 into 100 gives 1000000000 which divided by 100 + 7, that is, by 107, the quotient is 9345794, which is the number answering to the first year in the Table of 7 *per cent*. For although there be a remain, after the division, of 42, yet because $\frac{42}{107} < \frac{1}{2}$ it is here neglected: But if the remain had happened $\frac{52}{107} > \frac{1}{2}$, then 1 being added to the quotient, it is 9345795, and is so much neerer the thing required.

Secondly, 9345794 into 100 gives 934579400 which divided still by 107. quotient is 8734386, and 98 remaining, but $\frac{98}{107} > \frac{1}{2}$, therefore adding 1 to the quotient, it is 8734387, for the number answering to the second year in the Table of

of 7 *per cent*. After this manner are all the Tables made.

Use of the Tables.

This shall be shewed in a few Examples.

Example 1. *Interest profitable.*

If 100 *li.* give 6 *li.* for one year, what shall 500 *li.* give for 17 years, principall and Interest?

The Rule.

Multiply the Principall by the Radius, the Product is 5000000000, *which divide by* 3713643 (*which is the number answering to* 17 *years in the Table of* 6 per cent.) *the quotient will be* 1346 $\frac{1444522}{3713643}$ *of Pounds.* Which is the just sum of 500 *li.* with all its compound interest, at 6 *per cent.* for 17 years, and reduced it is 1346 *li.* 7 *s.* 8 $\frac{1}{2}$ *d.* the like way of working will effect any question of this nature, which exceeds not the Tables in time or rates.

NOTE.

It may here be noted, that if 1346 *li.* 7 *s.* 8 $\frac{1}{2}$ *d.* were due to be received 17 years hence, it is, or may be called equivalēt to the receiving of 500 *li.* in hand, that is, such a reversion is worth 500 *li.* in ready money.

And therefore by inversion of the former Rule, may the Rebatement, or Interest damageable be found.

Example 2. Interest damageable.

If there shall be 1000 li. due at the end of 21 years, and money run at 8 *per cent.* to be accordingly rebated, how much is this worth in ready money?

The Rule.

In the Table for 8 *per cent.* finde the number answering to 21 years, which is 1986557, multiply this by the principal, the product is 1986557000, which divided by radius the quotient is $198\frac{6557}{10000}$. pounds.

That is, 198 li. 13 s. $1\frac{1}{2}$ d. which is the thing required.

Probl. 1.

If 1000 li. be to be paid at the end of 7 years and 500 li. more 2 years after that, what shall both these be worth, for borne till the end of 12 years? at 6 *per. Cent.*

See (by the Rule belonging to Example 2.) First, what each of them is worth at the end of their proper termes,

The first is,	665,0571000
The second	295,9492500
In all	961,0063500

Which is all that both are worth in ready money.

Then secondly, seeke (as before) what 961, 0063500 li. ready money is worth 12 years hence, rebating interest of 6. *per. Cent.* It will come to about 478, 4778100 li. for the thing required.

Prob.

Probl. 2.

If there be due in ready money 500 *li.* which at the end of 20 years will increase, and be 2330, 477015 *li.* what is the rate of the interest here?

Take the fift part of the Number, (because the tables are made for 100 *li.*) which is 466,095403. and in the first table look for it against the number 20. it will be found in the rate of 8. *per. Cent.* and such is the interest.

Or in the third table.

Say, 2330,477015′ 10000000″ 500′ 2145482″.

Which last *viz.* 2145482. being looked for in the third table, will be found over against the year 20 under the title of 8. *per Cent.* which shews againe that the rate of the interest is 8. *per. Cent.*

And herein the Probleme is not onely cleared; but the use of both tables exemplified.

Prob. 3.

In like sort, if the interest, years, and totall increase be given, to find the principal. As if one receive 1000 *li.* for compound interest at the rate of 10. *per. Cent.* for 7 years; how much was the principal summe?

See in the third table in the rate of 10. *per. Cent.* what number answers to 7 years; it will be found 5131582. which being subtracted from 10000000 there rests 4868418. And then say, 4868418′ 5131582″ 1000′ 1054,269428″. That is, the principal was 1054 *li.* 1 *s.* 1 *d.* and a little more, which we omit.

And

And the same sum will be found if one use the first Table, where the interest of 100 *li.* for 7 years at 10 *per cent.* is 94,871710, for then 94,871710′ 100″ 1000′ 1054 *li.* 1 *s.* 1 *d*″. and a little more as before.

Probl. 4.

Or the rest being given, and the time required, As if there be 1000 *li.* due at the end of some years, and the Creditor instead of it takes 100 *li.* ready money, rebating compound interest at 8 *per cent.* at the end of what years was this at first payable?

Say 1000′ 10000000″ 100′ 1000000″. Which fourth proportionall number being (as neer as may be) sought for in the third Table under 8 *per cent.* will be found to fall neer 30 years: that is, the time here required is almost 30 years.

Prob. 5.

If 1000 *li.* be due at the end of 4 years, and the parties agree to have it paid at four yearly paymēts, that is 250 *li.* (rebating 6 *per cent.*) at the end of every year, how much is to be paid at each time?

1 Look in the third Table under 6 *per cent.* for 3 years, against that the number 8396192 stands, which last being multiplyed by 250, and after the product divided by Radius, that is by 10000000, the quotient is 209,9048000 *li.* that is, 209 *li.* 18 *s.* 1 *d.* for the first yearly payment.

2 In the same Tables against 2 years is found the number 8899964 which used in all respects like that against 3 years already done, the quotient

ent wil be 222,499 1000, that is, 222 *li.* 9 *s.* 11 *d.* for the second.

3 And against 1 year is 9433962, which being multiplyed like the two former, the quotient is 235,8490500, or 235 *li.* 16 *s.* 10 *d.* for the third payment.

4 Lastly, Must be the full fourth, *viz.* 250 *li.* for that being not paid otherwise then in due time, suffers no Rebatement.

Prob. 6.

If there be a Reversion of a Lease or Annuity of 100 *li. per Annũ*, and for 11 years to come at the end of 14 years, what is this worth in ready money? money being at 6 *per cent.*

1 Adde 11 to 14, it makes 25 years, and look in the second Table for 6 *per cent.* against 25 years there stands 5486, 7, 2 *d.* likewise against 14 years is 2101, 9, 2 *d.* the difference is 3384, 18, 0 *d.* which cannot be received till the end of 25 years. Therefore in the third Table for 6 *per cent.* against 25 years, finding the borrowed number 2329986, multiply it (as hath been lately shewed) by 3384 *li.* 18 *s.* 0 *d.* that is by 3384$\frac{9}{10}$ *li.* the product divided by 10000000 is 789,4672624 That is reduced 789 *li.* 9 *s.* 4 *d.* And so much it is worth in ready money, and the *Probl.* is solved.

All cases cannot be instanced, if the question happen to be like none of these, yet the Reader by his own judgement may (without doubt) resolve it by some of these Tables, to illustrate the use of which, was chiefly my end in putting and resolving the precedent demands

A

A Table of the Suns Declination

Dayes	Janu. South		Febru. South		Mar. South		April North		May North		Jun Nor	Dayes
1	21	49	13	56	3	35	08	26	17	58	23	1
2	21	39	13	36	3	11	08	48	18	13	23	2
3	21	29	13	16	2	48	09	09	18	28	23	3
4	21	18	12	55	2	24	09	31	18	43	23	4
5	21	07	12	35	2	00	09	53	18	57	23	5
6	20	56	12	14	1	37	10	14	19	11	23	6
7	20	44	11	53	1	13	10	35	19	25	23	7
8	20	32	11	32	0	49	10	56	19	38	23	8
9	20	19	11	10	0	26	11	17	19	51	23	9
10	20	06	10	49	0	02	11	37	20	04	23	10
11	19	53	10	27	0	N22	11	58	20	16	23	11
12	19	39	10	05	0	46	12	18	20	28	23	12
13	19	25	09	43	1	09	12	38	20	40	23	13
14	19	10	09	21	1	33	12	58	20	51	23	14
15	18	55	08	58	1	56	13	17	21	02	23	15
16	18	40	08	36	2	20	13	37	21	12	23	16
17	18	25	08	14	2	43	13	56	21	23	23	17
18	18	09	07	51	3	07	14	15	21	33	23	18
19	17	53	07	28	3	30	14	34	21	42	23	19
20	17	36	07	05	3	53	14	52	21	51	23	20
21	17	19	06	42	4	17	15	10	22	00	23	21
22	17	02	06	19	4	40	15	28	22	08	23	22
23	16	45	05	56	5	03	15	46	22	16	23	23
24	16	27	05	32	5	26	16	03	22	24	22	24
25	16	09	05	09	5	49	16	21	22	31	22	25
26	15	51	04	46	6	12	16	38	22	38	22	26
27	15	32	04	22	6	34	16	54	22	44	22	27
28	15	14	03	59	6	57	17	11	22	50	22	28
29	14	55			7	19	17	27	22	56	22	29
30	14	35			7	42	17	43	23	01	22	30
31	14	16			8	04			23	06		31

for the yeare 1654.

Days	July North	Aug. North	Septẽ. North	Octob. South	Novẽ. South	Decẽ. South
1	22 10	15 17	4 30	07 09	17 36	23 08
2	22 02	14 59	4 07	07 32	17 52	23 12
3	21 53	14 40	3 44	07 55	18 08	23 16
4	21 44	14 22	3 21	08 17	18 24	23 20
5	21 35	14 03	2 58	08 39	18 40	23 23
6	21 25	13 44	2 34	09 02	18 55	23 26
7	21 14	13 25	2 11	09 24	19 09	23 28
8	21 04	13 05	1 48	09 46	19 24	23 29
9	20 54	12 46	1 24	10 08	19 38	23 30
10	20 43	12 26	1 01	10 29	19 52	23 31
11	20 31	12 06	0 37	10 51	20 05	23 32
12	20 19	11 46	0 14	11 12	20 18	23 31
13	20 07	11 26	0 S 10	11 34	20 31	23 31
14	19 54	11 05	0 33	11 55	20 43	23 29
15	19 41	10 44	0 57	12 15	20 55	23 28
16	19 28	10 23	1 20	12 36	21 06	23 26
17	19 15	10 02	1 44	12 57	21 17	23 23
18	19 01	09 41	2 07	13 17	21 28	23 20
19	18 47	09 20	2 31	13 37	21 38	23 17
20	18 33	08 58	2 54	13 57	21 48	23 13
21	18 18	08 36	3 18	14 16	21 57	23 08
22	18 03	08 15	3 41	14 36	22 06	23 03
23	17 47	07 53	4 04	14 55	22 15	22 58
24	17 32	07 31	4 28	15 14	22 23	22 52
25	17 16	07 09	4 51	15 33	22 31	22 46
26	17 00	06 46	5 14	15 51	22 38	22 39
27	16 43	06 24	5 38	16 09	22 45	22 32
28	16 26	06 01	6 00	16 27	22 51	22 24
29	16 09	05 38	6 23	16 45	22 57	22 16
30	15 52	05 16	6 46	17 02	23 03	22 07
31	15 34	04 53		17 19		21 58

Postscript.

THe reason why in *Chap.* 15. I did not (as *Des Cartes*) continue the method to Surſolid Problemes, is becauſe the deſcription of ſuch curve lines, is not only difficult and laborious, but (as he confeſſeth) incommodious: And although he ſaith it is eaſie to finde a thouſand other ſorts of wayes, amongſt which ſome might be better, yet I conceive it is eaſier for any man to believe that *Des Cartes* having found one way might be allowed to ſay this, then for that other men to finde any one other way better and eaſier. And *Des Cartes* gives but only one Example, which is to finde four Means, and omits quinquiſection of angles, which might have been of ſome uſe, whereas the other is but of little.

For the principall uſe of two Means ſeems to be in doubling the Cube, or in making rectangle Parallelepipedons retaining any proportion given, whoſe three dimenſions alſo ſhall be proportionall: or the like of Solid Rhombi, or oblique

fique parallelepipedons, all which are bodies to be seen and handled, whereas four or more means to be so employed require bodies of four or more dimensions, of which we have yet no fancie.

Besides, *Galileo* in his Book called *Systema Mundi* (the beginning almost) seems to prove that there can be but three dimensions in nature. Neverthelesse, if any one happen to discover the Biquadraticall Body, he may then as well demonstrate that there can be but foure.

And yet more, seeing some Æquations of 6. dimensions require a circle to touch or cut a curve line in 6 points, which cannot be done but very obliquely, the method therefore here growes unusefull: and for septisection, and Æquations of 8 or more dimensions, it will be unsufficient.

If for all this, any one hath a minde to sursolids, his æquation, by some rules going before, or here following, must be reduced to this forme.

$$a^6 - ba^5 + ca^4 - da^3 + faa - ga + h = 0$$

In which it behoveth that the quantity called c, be greater then the square of $\frac{1}{2}b$.

In the section 6, of the first Rule of Chap. 4. it is shewed how all the false roots in any æquation may be made true.

Also in Rule the second of that Chap. it was shewed how to free the æquation from the second terme. There remaines three other Rules now of some use.

Rule

RULE 1.

To cause the known quantity of the third term to be greater then the square of the like in the second, and also to change the false roots to true ones, without causing the true ones to become false. *Encrease the true Roots by a quantity greater then any of the false Roots.* For sure it is possible to guesse such a quantity, although the false roots be unknown.

Example, in the æquation $+aaaa+ba^3 - ccaa - ddda + ffff = 0$

Put $e - b = a$ Then it will be

$$\left.\begin{array}{l} +e^4 - 2be^3 + bbee \\ \quad -2be^3 + 4bbee - 2bbbe \\ \quad +be^3 + bbee - 2bbbe + b^4 \\ \qquad -3bbee + 3bbbe + b^4 \\ \qquad +ccee - 2bcce + bbcc \\ \qquad\qquad -ddde - dddb \\ \qquad\qquad\qquad +ffff \end{array}\right\} = 0$$

That is

$$\left.\begin{array}{l} +e^4 - 3be^3 + 5bbee + ccee \\ \quad -b^3e - 2bcce - d^3e \\ +2b^4 + bbcc - dddb + ffff \end{array}\right\} = 0$$

And making

$5bb + cc = gg$ & $b^3 + 2bcc + d^3 = h^3$

And lastly, $2b^4 + bbcc + f^4 - bd^3 = llll$

Then it is

$+e^4 - 3be^3 + ggee - h^3e + llll = 0$

In which æquation all the Roots are true by *Sect.* 5. *Rule* 1. *Chap.* 4.

S And

And secondly, it is manifest that gg, which is the known quantity of the third terme, being equall to $5bb+cc$ is greater then $\frac{9}{4}bb$, which is the square of halfe the known quantity of the second term.

I have instanced in an Æquation of but 4 dimensions, for brevity sake, the work is true, or may be so, in those of six dimensions, which they that resolve sursolid Æquations will be put to.

RULE 2.

If yet any term of the Æquation be wanting, *Encrease the Root* e *never so little, and thereby all the places will be filled:* As practise will shew.

RULE 3.

If the Æquation have but 5 dimensions, it must be brought up to 6 as followeth. Let it be

$$+a^5-ba^4-dddaa+f^4a-g^5=0$$

In stead of it wrie

$$+a^6-ba^5-d^3aaa+f^4aa-g^5a=0$$

And make $e-q=a$, and the thing will be effected which was desired:

So if between two lines given b & c, and $c>b$, it be required to finde 4 meanes, putting a for the lesser mean, the Æquation will be $+a^5==bbbbc$, that is $+a^5=cb^4$, And the places which are empty may be filled up by the second Rule: and brought to 6 dimensions (if yet it be not so) by the 3d. Rule.

NOTE

[All Æquations whose dimensions are expressed

[illegible] numbers, can by no meanes be brought d[illegible] fewer dimensions by any artifice that can be used: and for this reason the invention of any even number of means is much harder then to finde an odd number of the like, so here where to finde foure meanes runs to an Æquation $aaaaa = bbbbc$, the Probleme is absolutely Sursolid: but to find five meanes is as easie almost, as to finde two, for the worke brings us to the Æquation $a^6 = cb^5$ which by the 5 Chapter may be brought downe to the Equation. $aaa = cdf$. or to $aaa = ddd$ if the Probleme be plaine, that is if it be $c' d'' f'''$ otherwise by making $df = gg$ it may be $aaa = ggc$. and the root a. found by a portion of a Parabola, as in the case of two means, *Chap.* 14.

Lastly, for quinquisection, the Radius being unity, put the whole subtense b. The subtense of the fift part required a. Then $+ a^5 - 5aaa + 5a - b = 0$. As is demonstrated by *Pitiscus*, in his making the sines, *Probl.* 7. and shall here need no proofe.

It will require some labour (as may be seen by *Chap.* 15. *Probl.* 1.) to bring this Equation to the forme required, for which purpose, first in stead of unity call Radius, r. And then it will be

$$+ a^5 - 5rra^3 + 5rrrra = 5rrrrb,$$

Afterward by *Rule* 3 it may be done.

And when this is done, the rem[illegible] work, as well Construction as Demon[illegible], according to the method proposed by *Des Cartes* may be done also, but the doing will be both tedious and intricate.

And therefore I shall no further prosecute the said Method, & for use, sectiõ of angles in generall may be done by the Rule of False Positions, as the said *Pitiscus* made his Canon of Sines. But the Canon of Sines, the late Prince of Mathematicians *VIETA*, vouchsafes to call *The Mathematicall Canon*. Neverthelesse, if the industrious Reader desires more exactnesse (I mean in Theorie) according to the former Method, or any other which he shall finde better for his purpose, he may proceed at pleasure,

A Rule for squaring binomiall surds, which should have been in Chap. 6. pag. 76.

Multiply the quantity to which the signe √ belongs, into the square of the Coefficient, and the product is the square required.

Example in Numbers.

If the square of 3√7 be required, multiply 7 into 9, the product is 63, the square required.

Or, If the square of 4√9 be demanded, 9 into 16 produceth 144, which is the square demanded: the like of all others. This needs no [illegible] See *Page* 77. And this shews that all such [illegible] are commensurable in power.

Deo Gloria.

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